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EFFECTS OF EXPONENTIALLY DECAYING SPATIAL PATTERNS ON THE PROBABILITY DISTRIBUTION OF ANOMALOUS VALUES

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ABSTRACT

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The nature of geochemical anomalies is discussed on a speculative basis. Anomalies are classified as independent or as additive, depending on their spatial relation to the background population. The additive anomalies, which are more commonly encountered in geochemical exploration, cannot be separated from the background, in a composite population, by extracting straight lines from curved cumulative frequency graphs, because the cumulative frequency of the distribution of the additive component of an anomaly plots as a curved line on normal and lognormal probability paper.

INTRODUCTION

Plots of cumulative frequency distributions on arithmetic or logarithmic probability paper are commonly used among exploration geochemists to estimate distribution laws and parameters of the data studied. Geochemical data are commonly conceived as consisting of two or more populations which normally have overlapping frequency distributions. One or more of the populations are usually thought to represent mineralization and are referred to as anomalous populations.

The cumulative frequency graphs of metal contents of geochemical samples are commonly curved and frequently exhibit a sigmoidal form on probability paper. This has been interpreted as being the result of a population of anomalous values, which has a normal or lognormal frequency distribution superimposed on a normally or lognormally distributed background population. However, the form of the frequency distribution of anomalous populations may well depart from the simple normal or lognormal model commonly used.

Much attention has been focused on the problem of extracting the constituent populations from composite data distributions. On probability paper straight lines have commonly been estimated from curves to represent the

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frequency distributions of the constituent populations (Tennant and White, 1959; Williams, 1967; Lepeltier, 1969; Bølviken, 1971; Parslow, 1974; Sinclair, 1974, 1976). In the present paper the author presents an argument that the cumulative frequency curves of metal contents originating in mineralizations do not, in most cases, plot as straight lines on normal or lognormal probability paper.

According to Lepeltier (1969) and Sinclair (1974) there are certain advantages in cumulating from high to low values when presenting cumulative frequency curves. The validity of this conclusion can be doubted because the cumulated frequencies are calculated for and plotted at the limits of the content intervals. If the frequencies of a set of data are cumulated in both directions and plotted on the same probability paper, the two resulting curves will be mirror images of each other against the 50% line. Therefore, there is no advantage to reversing the direction of cumulation.

NATURE OF GEOCHEMICAL ANOMALIES

Here geochemical anomalies are classified into independent anomalies and additive anomalies. The classification is based on the spatial relation between the background population and the anomalous population.

Independent anomalies

In an independent anomaly, as defined here, the total amount of an element in a geochemical sample has a common origin. In lithogeochemistry, for example, a sample may be taken either from a barren rock or from a mineralized rock. In a soil sample a metal may be totally derived from a barren rock or from a mineralized rock. In these cases it is possible to separate the geochemical samples and corresponding data into two areally separated populations, a background population and an anomalous population. The frequency distributions of the data of both populations may be approximately normally or lognormally distributed.

Additive anomalies

A vast majority of anomalies of interest in mineral exploration are formed by migration of elements from mineralized rocks into the surrounding environment. Geochemical sampling is commonly aimed at finding and studying these anomalies because their areal size is usually much larger than that of their source. The bulk of each sample in such an anomaly is barren material with an inherent content of each element which may be considered typical of the background population. Elements derived from mineralized rocks through weathering and geochemical migration are added to the background content in each sample within the anomaly.

In the present paper the additive anomaly is defined as the component which is added to the background in each sample, and which, as will be shown, cannot be extracted for separate study from the composite population by simple graphical means. The frequency distribution of additive anomalies are therefore evaluated in the present paper on a speculative basis.

Krumbein (1937) suggests, that certain characteristics of some sedimentary deposits vary exponentially as a function of distance. He presents several examples to illustrate this, including logarithmic decay of boulder fans with distance from the source. That is, when the contents in an anomaly are plotted against the distance from the source, the decreasing values plot as a straight line if one of the axes is logarithmically scaled. Morris and Lovering (1952) found that the contents of Pb and Zn in wall rocks decrease exponentially with the distance from lead-zinc deposits in the Tintic district of Utah. Beus and Grigorian (1977) use contents which decrease exponentially with increasing distance from the source as a model for their methods of evaluating epigenetic anomalies in the primary environment. Gillberg (1965) showed that anomalies in till decrease exponentially with the distance down-ice from the source. In stream sediments anomalies tend to decay rapidly close to the source, whereas, further downstream they slowly taper off to background contents. The speculations which follow are based on the assumption that in superficial material such as water, stream and lake sediments, soil, vegetation, air etc. anomalies originating in "point sources" such as mineralizations decay more or less exponentially with the distance from the source.

FREQUENCY DISTRIBUTION OF ADDITIVE ANOMALIES

To estimate the form of the frequency distributions of the contents in additive anomalies, hypothetical data were simulated for the following cases:

(A) Exponentially decaying anomaly trains were added to lognormally distributed background contents.

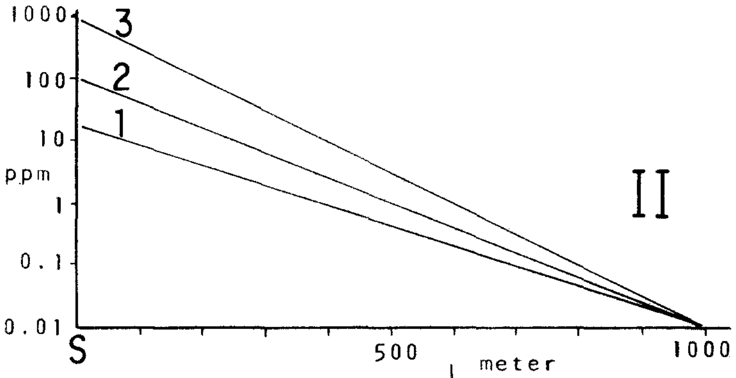
(B) Exponentially decaying anomaly trains were added to normally distributed background contents.

(C) A two-dimensional exponentially decaying anomaly was added to lognormally distributed background contents.

Case A

For 100 sites along 1000 m of a stream channel, contents of an element were drawn randomly from a hypothetical lognormal population. The resulting content profile is presented in part I of Fig. 1. The frequency distribution of these data is presented in the upper left corner of Fig. 2 (*A*). This population is used as a background value in the following discussion.

If a mineralized source is situated on the left-hand (up stream) end of the sampled section of the stream, an anomalous component would be added to each sample down stream. An additive anomaly would result. The amount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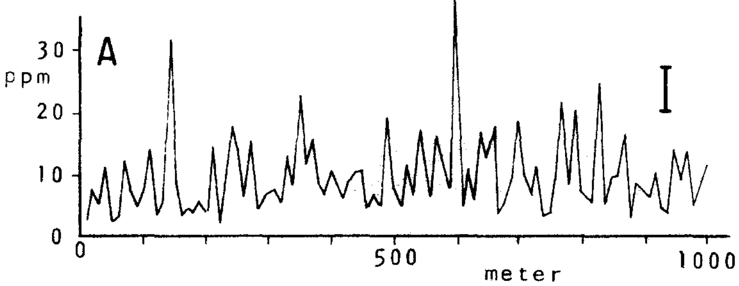
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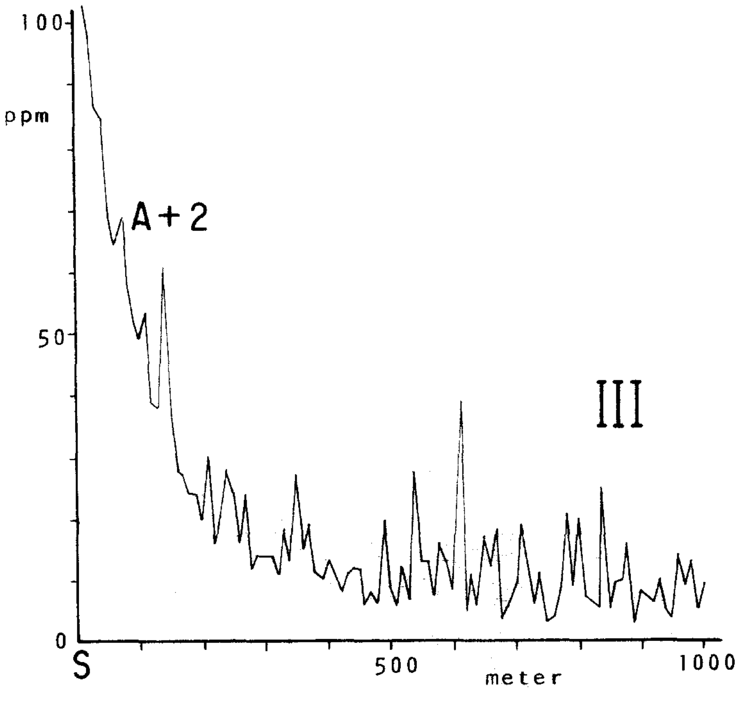
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of the additive component is assumed to decrease exponentially with distance from the source. In part II of Fig. 1 the content profiles of three idealized hypothetical additive anomalies are shown. Content profiles *1*, *2* and *3* represent anomalies with maximum contents of 20 ppm, 100 ppm, and 1000 ppm respectively at the source (S). Each one decays exponentially with distance from the source and is truncated 1000 m down stream at 0.01 ppm. In Fig. 1, part III, the anomaly profile *2* of part II is added to the background profile A of part I to result in the composite anomaly profile A+*2*. In real geochemical surveys composite profiles are encountered.

In the upper left corner of Fig. 2 the frequency distributions of the populations of Fig. 1 are shown. The background population (A) has a lognormal frequency distribution. The density curves of the additive anomalies 1 and 2 (3 is not shown) rise until they are truncated at the lowest content. In the case of sampling points with uniformly distributed distances from the maximum content (a) of the anomalous component at the source, to the minimum content (b) of the anomalous component at the distance (d) from the source, the frequency function for the concentration (t) of the anomalous component is:

$$f(t) = \frac{1}{\ln(a/b)t} \qquad b \leqslant t \leqslant a$$

This formula can be directly derived by variable transformation because the assumed transformation function $g\ (x) = a\ \mathrm{e}^{\alpha x}$ ($\alpha = (1/d \ln b/a)$ of the sampling distance is monotonically decreasing (Hald, 1952). For an anomaly with no truncation, the curve would approach the frequency axis asymptotically, which would give a hyperbolic form. Histogram A+*2* (Fig. 2) represents the data of the content profile in part III of Fig. 1, that is, an additive anomaly (*2*) superimposed on a lognormally distributed background population (A). On addition, the skewness of the frequency distribution is increased but no second mode appears.

The cumulative frequency curves of the additive anomalies are sigmoidal on logarithmic probability paper (curves *1*, *2*, and *3* in Fig. 2). When each of the anomalies is added to the background population (curve A) the sigmoids A+*1*, A+*2*, and A+*3* result, respectively.

In a real case an additive anomaly cannot be traced down to 0.01 ppm but is obscured by the background variation up to much higher content levels. In part III of Fig. 1 the visually detectable anomaly is only some 300—400 m long. In the right-hand part of Fig. 2 the cumulative frequency curves of

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Fig. 1. I. Content profile of a hypothetical background population (A) of 100 points along 1000 m of a stream. II. Content profiles of hypothetical additive anomalies *1*, *2* and *3* along the stream with maximum values of 20, 100, and 1000 ppm respectively at the up stream end over the source (S) and with exponentially decreasing contents with increasing distance from the source. III. Composite content profile A+*2* derived by addition of anomaly *2* to the background contents of profile A.

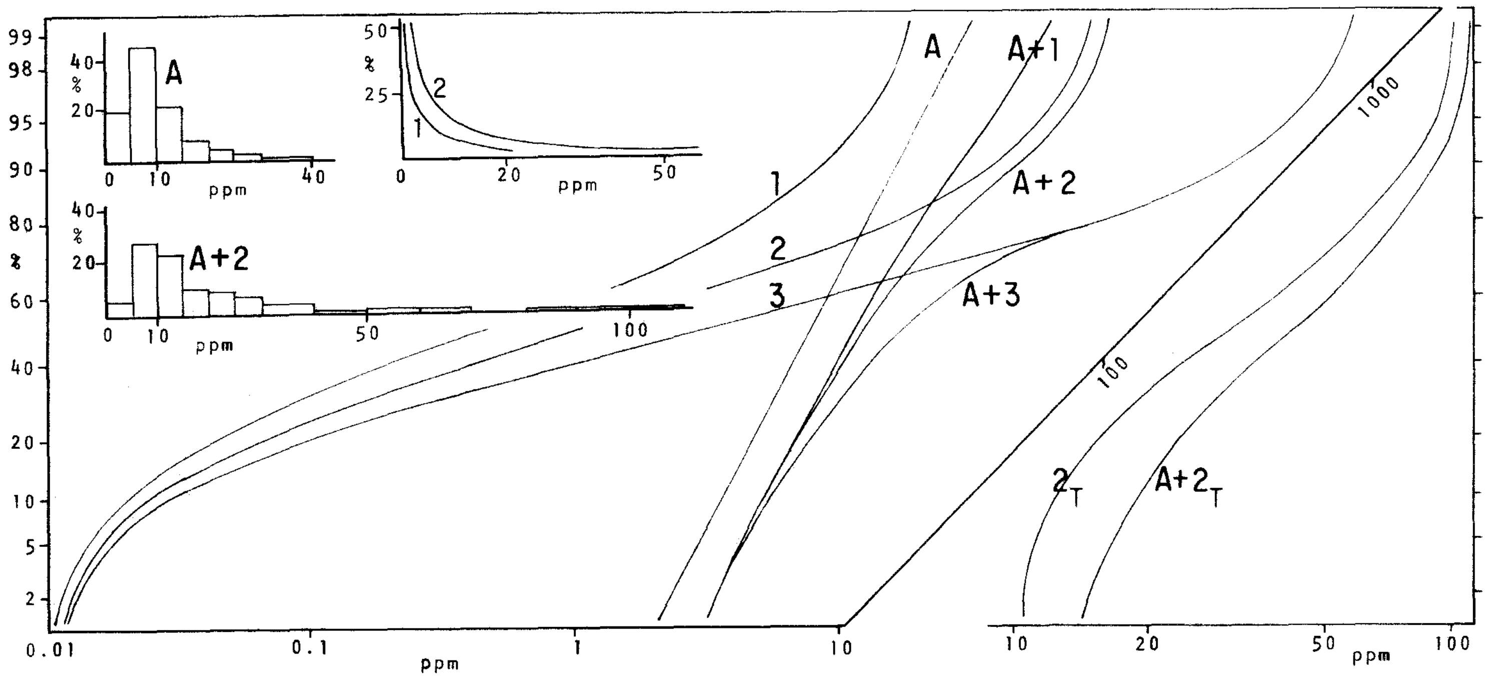

Fig. 2. Upper left corner: Frequency distributions of the hypothetical lognormally distributed background population *A* of Fig. 1, of the hypothetical additive anomalies *1* and *2* of Fig. 1, and of the composite population of the background *A* and the additive anomaly *2* (*A+2*) of Fig. 1. Middle part: Cumulative frequency curves of the hypothetical additive anomalies *1*, *2*, and *3* of the hypothetical background population *A*, and of the composite populations of the background *A* and the additive anomalies *1* (*A+1*), *2*(*A+2*), and *3*(*A+3*). Right part: Cumulative frequency curves of the hypothetical additive anomaly *2*, truncated at 10 ppm at the lower end (2_T), and of the composite population of the background A and the truncated anomaly $2_T(A+2_T)$.

this detectable portion of the anomaly are shown. Curve 2_T represents the frequency of the hypothetical additive anomaly *2* as truncated at 10 ppm. Curve $A+2_T$ represents the frequency distribution of the composite population consisting of the truncated anomaly superimposed on the lognormally distributed background population (*A*). Again the curve representing the composite population is a sigmoid.

Case B

In Fig. 3 the anomalies *1*, *2*, and *3* of Fig. 1 have been added to hypothetical background contents having a normal frequency distribution to give the curves and histograms. The histograms of the background values (*B*) and of the background plus the additive anomaly 2 (*B+2*) are shown in the upper left corner of the figure. The anomaly, when added to the background, skews the distribution but does not give a second mode. The cumulative frequency graph B of the normally distributed background population is curved because it is plotted on logprobability paper. When the weak anomaly (*1*) is added to the background the resulting cumulative frequency curve is approximately straight (*B+1*). Anomalies *2* and *3*, when added to the normally distributed background population, give curved cumulative frequency graphs (*B+2* and *B+3*). In a real case, where sampling and analytical errors scatter the frequency curve, *B+1* would probably be interpreted as a straight line representing a lognormal distribution. Curves *B+2* and *B+3* would be considered as having approximately sigmoidal form representing a composite of two lognormally distributed populations or they could even be interpreted as the result of the mixing of three populations because each curve has two inflection points.

Case C

To evaluate the frequency distribution of a two-dimensional additive anomaly a hypothetical equidimensional hydromorphic anomaly has been simulated. In the upper left corner of Fig. 4 a sampling grid is laid over an additive anomaly which has a maximum content of 100 ppm above the source (*S*). The contents of the anomaly decrease exponentially with the distance from the source in all directions. The cumulative frequency curve of the additive anomaly truncated at 0.01 ppm has a sigmoidal form on logarithmic probability paper (Fig. 4, curve 2_2). When the anomaly is added to a hypothetical lognormal background population with the same frequency distribution as histogram *A* in Fig. 2, the cumulative frequency curve of the resulting composite population froms a bent line with two straight sections (Fig. 4, curve $A+2_2$).

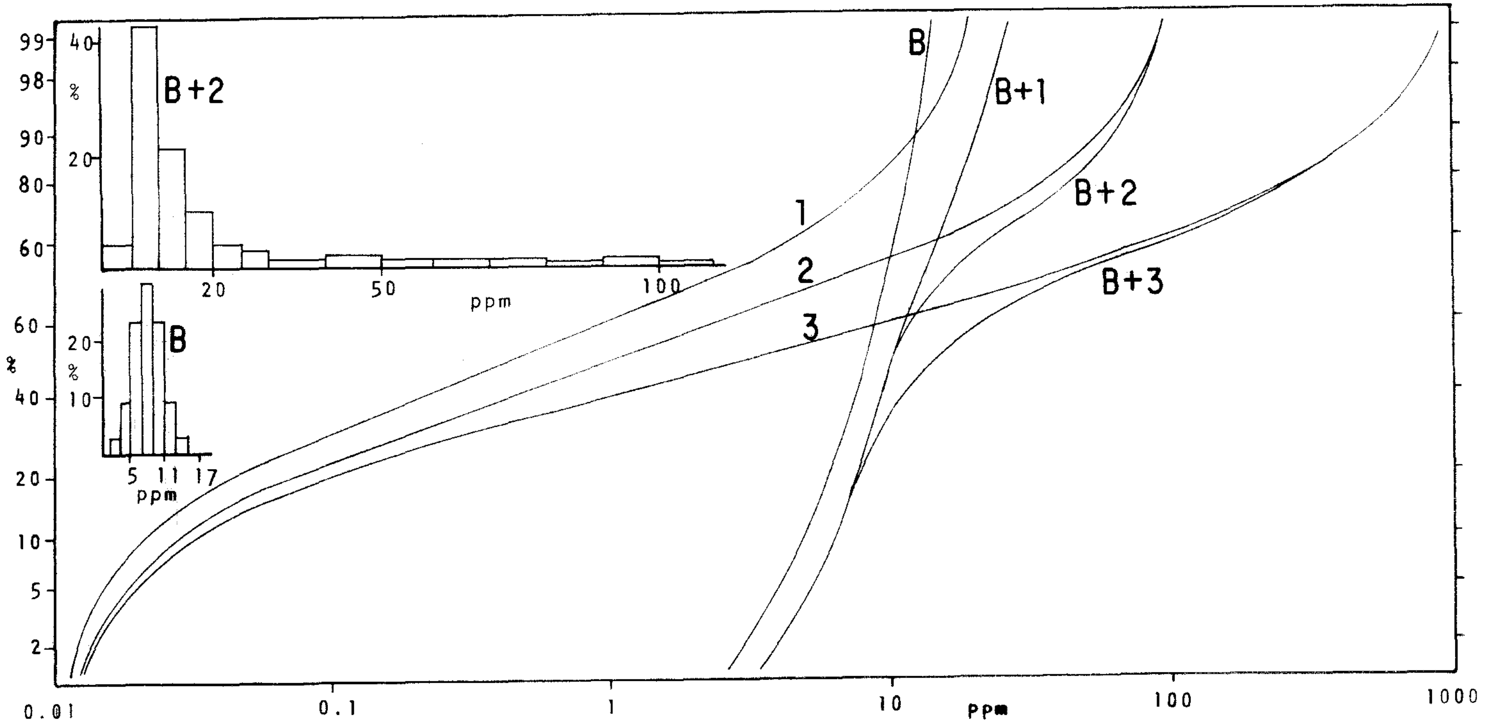

Fig. 3. Left part: Frequency distributions of a hypothetical normally distributed background population *B*, and of the composite population *B* + *2* of the background *B* and the hypothetical anomaly *2* of Fig. 1. Middle part: Cumulative frequency curves of the hypothetical additive anomalies *1*, *2* and *3* of Fig. 1, and of the composite populations of the background *B* and the additive anomalies *1* (*B* + *1*), *2*(*B* + *2*), and *3*(*B* + *3*).

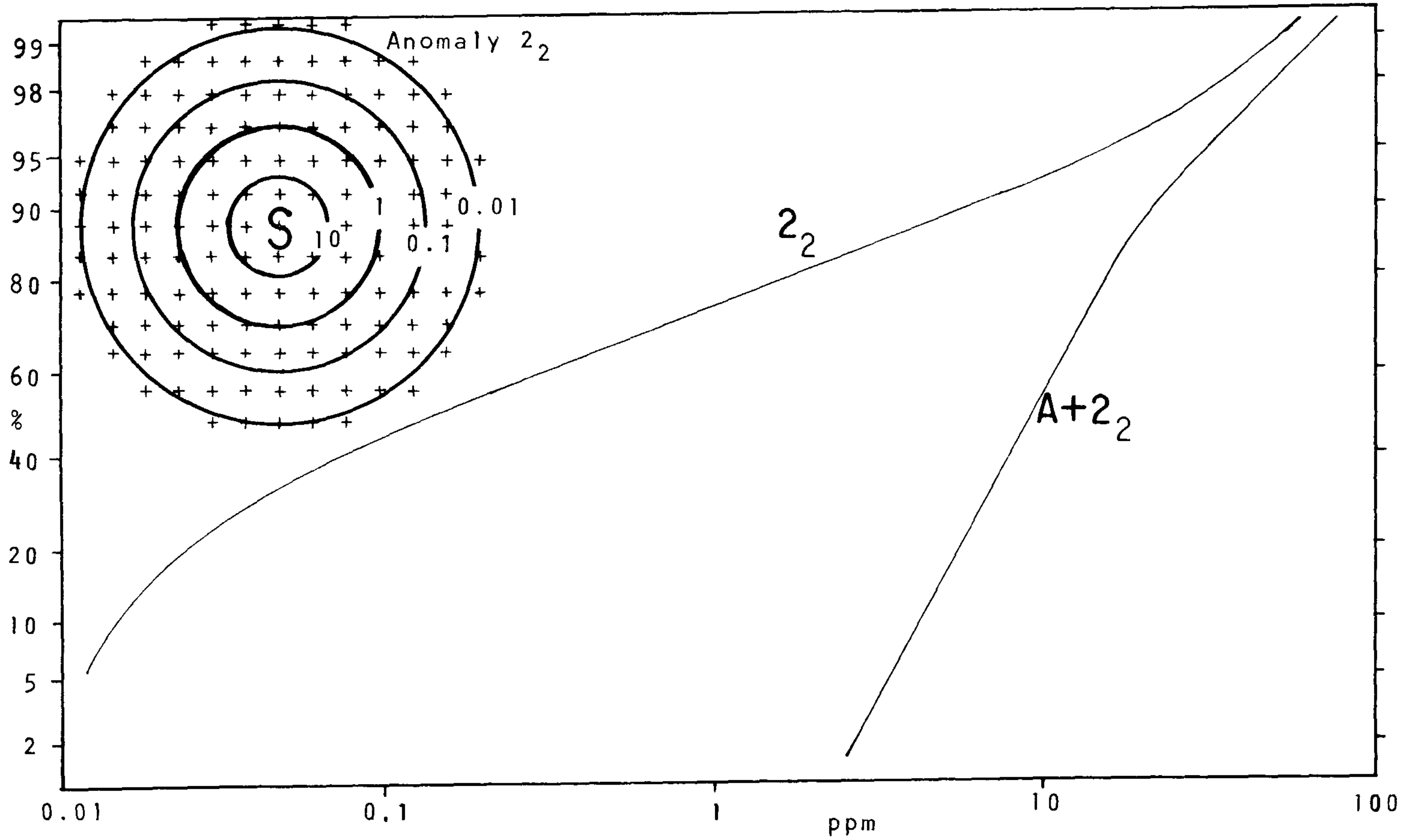

Fig. 4. Sampling sites (+) over an hypothetical two-dimensional additive anomaly (2_2) with 100 ppm over the source (*S*) and with the concentric isocontent curves 10, 1, 0.1 and 0.01 ppm forming an exponentially decreasing pattern with the distance from the source. Cumulative frequency curves of the additive two-dimensional anomaly 2_2, and of the composite population $A + 2_2$ of a lognormally distributed background *A* and the additive anomaly.

DISCUSSION

With the hypothetical examples above, the author tries to show that anomalous subpopulations of geochemical data do not necessarily have an approximately normal or lognormal frequency distribution. Anomalies displaced from their sources, which are very common, especially in the secondary environment, do not decrease linearly with the distance from the sources but normally decay rapidly close to the source and taper off slowly to background contents further away. Anomalies having contents which decay exponentially with the distance from the source have been evaluated on a speculative basis above. Outside the source the anomalous contents have been added to the inherent background contents of the material sampled. The frequency curve of these additive contents of a regular sampling grid is a hyperbolic, which plots as a sigmoid on logarithmic probability paper. The cumulative frequency curve of the composite population (background + additive anomaly) may also plot as a sigmoid on logarithmic probability paper. For two-dimensional anomalies the cumulative frequency curve of the composite population may have only a change of slope in the upper end. In all these cases it is not possible to partition the cumulative frequency curves into straight lines representing the constituent populations.

In the examples cited only one anomaly at a time was added to the background. Normally in a survey area there are a number of additive anomalies with varying maximum contents. The contents of these anomalies may be added into one population. Tentative work has indicated that even in this case, the cumulative frequency graph of the composite additive component forms a sigmoid if the anomalies decay exponentially.

The author is fully aware of the fact that an ideal exponentially decaying anomaly pattern is only one special case of all the possible patterns which may exist. In reality a population of data (contents) apparently is composed of a variety of subpopulations, each one following a specific distribution law and included in the composite population in its own specific way. Under these circumstances of additive anomalies it is difficult to believe that a simple graphical technique could be developed for routine use in partitioning the composite populations.

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