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Damage identification in bars with a wave propagation approach: Performance comparison of five hybrid optimization methods

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Abstract. The formulation and solution of the inverse problem of damage identification based on an one-dimensional wave propagation approach are presented in this paper. Time history responses, obtained from pulse-echo synthetic experiments, are used to damage identification. The identification process is built on the minimization of the squared residue between the synthetic experimental echo, obtained by using a sequential algebraic algorithm, and the corresponding analytical one. Five different hybrid optimization methods are investigated. The hybridization is performed combining the deterministic Levenberg-Marquardt method and each one of the following stochastic techniques: The Particle Swarm Optimization; the Luus-Jaakola optimization method; the Simulated Annealing method; the Particle Collision method; and a Genetic Algorithm. A performance comparison of the five hybrid techniques is presented. Different damage scenarios are considered and, in order to account for noise corrupted data, signals with 10 dB of signal to noise ratio are also considered. It is shown that the damage identification procedure built on the Sequential Algebraic Algorithm yielded to very fast and successful solutions. In the performance comparison, it is also shown that the hybrid technique combining the Luus-Jaakola and the Levenberg-Marquardt optimization methods provides the faster damage recovery.

Keywords: Structural damage identification, acoustic wave propagation in solids, sequential algebraic algorithm, stochastic optimization methods, hybrid optimization methods

1. Introduction

Structural health monitoring (SHM) and damage identification (DI) are prime concerns in the realm of civil, mechanical and aerospace engineering. They represent essential issues to determine the safety and reliability of their systems and structures.

Different nondestructive SHM and DI approaches are proposed in the literature. Most of them, however, are built on changes in the vibration characteristics of the structures under concern [1,18]. The basic idea of these approaches

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is that the modal properties (frequencies, mode shapes and damping ratios) are functions of the physical properties of the structure (mass, stiffness and damping) and, therefore, changes due to damage in the physical properties will be reflected in the modal ones, which, can be measured and used to infer about the damage state.

Although the vibration-based damage identification approaches have been successfully applied to practical problems [10,25], it is well known that small defects may yield to excessively small or even no effects on the modal properties of the structure, making the damage identification a more difficult task. Damage identification methods built on the acoustic wave propagation approach, on the other hand, are highly sensitive to changes in local dynamic impedance such as those caused by small defects [3,22]. Besides the higher sensitivity to small defects, the wave propagation based approaches are directly defined in the time domain and, therefore, differently from most of the vibration based methods, they do not require any signal processing for compressing the acquired data to the modal space, which inherently results in some loss of information.

Applications of the elastic wave propagation approach in the framework of structural damage identification are recently reported in the literature. Damage identification in laminated beams are reported in [8,13]. Fatigue damage assessment is presented in [2,14]. Applications in damage assessment and structural health monitoring in aircraft fuselages are reported in [26,27]. The modeling of the acoustic wave propagation phenomenon in the frequency domain by the Espectral Element Method for damage identification purposes has been extensively considered. Applications are reported for beam type structures [7,30], plates [29,31] and L-join elements [28].

The main goal of this research is to study the inverse problem of damage identification in bars within the framework of acoustic wave propagation approach. In a previous work [23], the direct problem of one-dimensional acoustic wave propagation was addressed by considering the Sequential Algebraic Algorithm (SAA). The inverse problem of damage identification was then casted as a minimization one, in the time domain, and the Particle Swarm Optimization was considered for minimizing the squared difference between the experimental (or synthetic) echo and the one predicted by the SAA. An experimental validation of the SAA model was also presented.

In the present work, aiming at solving the inverse damage identification problem, five hybrid optimization techniques are considered. The hybrid techniques are obtained by combining a stochastic optimization method with the deterministic method of Levenberg-Marquardt (LM). The main interest in using hybrid techniques relies on the fact that, in spite of being a fast technique, the results provided by the LM method strongly depend on an arbitrary parameter, the relaxation factor. For an unsuitable choice of this parameter the result of the LM method may even diverge. On the other hand, the stochastic optimization methods usually yield to a relatively large residual error in the identification or to a prohibitively large computational cost. The combination of the two techniques, however, provides damage assessment results with greater performances and better accuracy. The stochastic methods considered for hybridization with the LM method are: the Particle Swarm Optimization method (PSO) [5]; the Luus-Jaakola optimization method (LJ) [9]; the Simulated Annealing (SA) method [6]; the Particle Collision Algorithm (PCA) [19]; and a Genetic Algorithm (GA) [4].

For each damage scenario, a pulse-echo synthetic experiment is performed and the excitation and corresponding response are considered in the damage identification procedure. In this work, the damage state is described by the cross section area $A(x)$ of the bar, were x is the position variable along the bar. Therefore, the damage identification is performed by minimizing, with respect to $A(x)$, the squared norm between the experimental (synthetic) echo and the predicted one.

The paper is organized as follows. In Section 2 a summary of the theoretical wave propagation direct model is presented. It is almost the same as presented in [23], but discussed again here for the sake of completeness. In Section 3 the adopted optimization procedures are displayed. For each one of the adopted techniques a short summary is shown. Section 4 presents the damage scenarios addressed in the present paper. In Section 5 the damage identification results with noiseless data is presented. Section 6 is devoted to the results of damage identification with noisy data. The performance comparison of the considered hybrid methods is discussed in Section 7. Finally, the main conclusions are presented in Section 8.

2. Mathematical formulation of the direct problem

The one-dimensional longitudinal acoustic wave propagation in a non-homogeneous slender bar can be described by the hyperbolic second-order differential equation [17]

$$\sigma_{tt} - c^2 \left[\sigma_{xx} + \left(\frac{A'}{A} - \frac{\rho'}{\rho} \right) \sigma_x + \rho \left(\frac{A'}{\rho A} \right)' \sigma \right] = 0, \tag{1}$$

where $\sigma(x,t)$ is the longitudinal stress field, depending on the position x and time t, $A(x)$ is the bar cross section area, $\rho(x)$ is the bar density, both depending on the position x, c is the longitudinal acoustic wave speed, the prime stands for total derivative, and the subscripts, as usual, represent partial derivatives.

The general D'Alembert solution for Eq. (1) cannot be obtained in a closed form. However, it can be shown [21], that Eq. (1) can be written in an alternative form, in the characteristic plane (r, s), as the following system of first-order equations:

$$\begin{aligned} &U_r + \frac{\dot{Z}}{4Z} U = 0; \\ &V_s - \frac{\dot{Z}}{4Z} V = 0, \end{aligned} \tag{2}$$

where $Z = \rho c A$ is called the *generalized acoustical impedance*, $U(r,s)$ and $V(r,s)$ are, respectively, the *progressive* and *regressive* stress wave components traveling along the characteristic plane, defined as

$$\begin{aligned} &r = t + \tau; \\ &s = t - \tau, \end{aligned} \tag{3}$$

and the dot stands as derivative with respect to the independent variable τ, the *travel time*, defined as

$$\tau(x) = \int_0^x \frac{d\xi}{c(\xi)}. \tag{4}$$

Equations (2) are a *compact and uncoupled* pair of first order differential equations that describes the longitudinal acoustic wave propagation phenomenon in a more convenient way. To integrate it, boundary conditions in the (r, s) plane must be provided, corresponding to the physical situation under concern. Let us, for instance, consider the probing of a medium, $x \geqslant 0$, by a pulse excitation at $x = 0$. Assuming also the Sommerfeld radiation hypothesis [16], the boundary conditions can be stated as:

$$U(s,s) = F(s) = f(t); \quad V(r,0) = 0, \tag{5}$$

where $f(t)$ is the incident longitudinal stress being applied at the boundary $r = s$ ($x = 0$) and the second equation ensures that there is no disturbance at $s \leqslant 0$ ($t \leqslant x/c$). Note that $f(t)$, being the longitudinal stress at the physical boundary $x = 0$, corresponds to $U(s,s)$, a *progressive* wave component. Analogously, the echo observed at $x = 0$, due to the inhomogeneity caused by the damage, will be the output signal $g(t) = V(s,s)$, a *regressive* wave component.

Assuming now that the bar under study has lenght l and is discretized in n sections with equal length $\Delta x = c\Delta t$, so that $l = n\Delta x$. The (known) discrete incoming pulse is written then as

$$F_j = f\big(2(j-1)\Delta t\big), \quad j = 1, 2, \ldots, N, \tag{6}$$

and the discrete outgoing echo is

$$G_j = g(2j\Delta t), \quad j = 1, 2, \ldots, N, \tag{7}$$

where $N\Delta t \leqslant \Delta T$ is the total time interval under consideration.

It can be shown that Eq. (2), with the boundary conditions given in Eq. (5), have, after the discretization given in Eqs (6) and (7), the following algebraic solution for the echo [21]

$$G_j = \sum_{k=1}^{j} \left(R_k + \sum_{p=1}^{k-2} Q_k^p \right) F_{j-k+1}, \quad j = 1, 2, \ldots, N \tag{8}$$

where the polynomials Q_k^p have the general recursive formula

$$Q_k^p = R_{k-p} \left[\frac{Q_{k-1}^p}{R_{k-p-1}} - R_{k-p-1} \left(R_{k-1} + \sum_{l=1}^{p-1} Q_{k-1}^l \right) \right], \quad k = 1, 2, \ldots, \quad p = 1, 2, \ldots, k-2. \tag{9}$$

In Eqs (8, 9), R_i stands for the *reflection coefficient* at the $i\text{-}th$ layer of the medium, defined as

$$R_i = \frac{Z_i - Z_{i-1}}{Z_i + Z_{i-1}}, \quad i = 1, 2, \ldots, n. \tag{10}$$

where $Z_i = \rho c A_i$ is the discretized generalized acoustical impedance of the medium. Since ρ and c are assumed as constant, the reflection coefficient can be written as

$$R_i = \frac{A_i - A_{i-1}}{A_i + A_{i-1}}, \quad i = 1, 2, \ldots, n. \tag{11}$$

The mathematical procedure, in the direct wave propagation approach, consists then in the following steps. The medium, with a nominal cross-section area A_0 and nominal generalized acoustical impedance Z_0, is discretized into n elements. Then, the reflection coefficients are computed by Eq. (11). In the sequel, the polynomials Q_k^p are calculated from Eq. (9). Finally, the output echo is computed from Eq. (8). The described technique is called *Sequential Algebraic Algorithm* (SAA).

It is worth stressing that the mathematical model above provides an original algebraic formula to solve the direct acoustic wave propagation problem. It also permits, in the identification procedure, to identify one parameter per step. As it will be seen in the damage assessment results, the number of parameters that may be updated in the identification is significantly larger than what is usually found in an optimization processes.

It is worth noting that it is not necessary to consider an infinite or even a semi-infinite medium. The echoes will be observed in the interval ΔT, that means, $t \in (0, 2l/c)$. This means that the echo originated by the right end of the bar – whatever is its boundary condition – is irrelevant for the analysis.

3. Identification procedure

For damage identification purposes, the bar is spatially discretized into n sections with equal length, so that the cross section area profile $A(x)$ is approximated by sectionally constant values $A_i, i = 1, 2, \ldots n$.

Defining the vector

$$\mathbf{A} = \{A_1, A_2, \ldots, A_n\}, \tag{12}$$

the damage identification problem under concern may be posed as a finite dimensional minimization one as follows.

$$\min_{\mathbf{A}} E, \tag{13}$$

where the functional E is the squared norm of the residue vector $\mathbf{r}(\mathbf{A})$, which is defined as

$$\mathbf{r}(\mathbf{A}) = \left\{ \begin{array}{c} G_1(\mathbf{A}) - \overline{G}_1 \\ G_2(\mathbf{A}) - \overline{G}_2 \\ \vdots \\ G_N(\mathbf{A}) - \overline{G}_N \end{array} \right\}, \tag{14}$$

where N is the number of data in the time series considered for the identification process, $G_j(\mathbf{A})$ is the echo predicted by the SAA model at the time instant t_j, and $\overline{G}_j$ is the corresponding experimental (or synthetic) echo. Therefore, from Eq. (14), one has

$$E = \sum_{j=1}^{N} \left[G_j(\mathbf{A}) - \overline{G}_j \right]^2. \tag{15}$$

3.1. Hybrid optimization methods

Aiming at solving the damage identification problem defined in Eq. (13), the present work considers hybrid optimization techniques, which were obtained through the combination of the deterministic Levenberg-Marquardt (LM) method and each one of the following stochastic methods: The Particle Swarm Optimization (PSO) method, the Luus-Jaakola (LJ) method, the Particle Collision Algorithm (PCA) method, the Genetic Algorithm (GA) method, and the Simulated Annealing (SA) method.

Recently, hybrid approaches, coupling stochastic methods and the LM one have been successfully used for solving inverse problems of parameter estimation [20] as SA-LM (Simulated Annealing and Levenberg-Marquardt) and GA-LM (Genetic Algorithms and Levenberg-Marquardt). Other hybrid strategies combining stochastic and deterministic methods are reported in the literature [24].

In general, stochastic methods require an extremely large number of evaluations of the cost function to achieve the global minimum, but the convergence of the solution to a local minimum can be avoided. The deterministic Levenberg-Marquardt method, on the other hand, presents faster convergence to the global minimum if a good initial guess for the parameters to be identified is available. Therefore, in each hybrid technique considered in the present work, the corresponding stochastic method is applied to the problem with the aim at obtaining a good initial guess for the Levenberg-Marquardt method. Hence, each hybrid method tries to keep the best features of the stochastic and the deterministic methods, so that the global minimum is likely to be achieved and the number of iterations required to obtain the minimum is greatly reduced. In the present work, a very brief description of the general idea of the addressed methods is presented. For more details, a basic literature is furnished.

3.1.1. The particle swarm optimization method

The PSO is a population based search algorithm. It was inspired from natural behavior of animals [5]. The population contains a set of individuals, or agents, referred to as *particles*, where each one represents a possible solution for a given optimization problem. These particles are, in general, randomly initialized. During the PSO process, each particle, based on a given evaluation criterion, updates its own position with a certain speed, which is computed based on both the best experience of the particle itself and that of the entire population. This update process is repeated for a certain number of generations. The update process stops either when the objective is achieved or when the maximum number of generations is reached.

Step 1. The number m of particles in the swarm is specified. The position and velocity of the particle i at the time step k are represented, respectively, by $\mathbf{x}_{i,k}$ and $\mathbf{v}_{i,k}$. The initial position and velocity of each particle is randomly generated as

$$\mathbf{x}_{i,0} = \mathbf{x}_{\min} + r(\mathbf{x}_{\max} - \mathbf{x}_{\min}), \tag{16}$$

$$\mathbf{v}_{i,0} = \frac{\mathbf{x}_{\min} + r(\mathbf{x}_{\max} - \mathbf{x}_{\min})}{\delta t}, \tag{17}$$

where r is a random number within the interval $[0, 1]$, $\mathbf{x}_{\min}$ and $\mathbf{x}_{\max}$ defines the search domain and δt is the time discretization. Here, δt was adopted as 1 and a new time instant corresponds simply to a new iteration of the algorithm;

Step 2. Each particle velocity is updated as

$$\mathbf{v}_{i,k+1} = c_1\, \mathbf{v}_{i,k} + c_2 r_1(\mathbf{p}_{i,k} - \mathbf{x}_{i,k}) + c_3 r_2(\mathbf{g}_k - \mathbf{x}_{i,k}), \tag{18}$$

where c_1 is the inertia weight, which controls the impact of the previous velocity on the current one; c_2 and c_3 are positive constants, called cognitive and social parameters, respectively; r_1 and r_2 are two random numbers in the range $[0, 1]$, whose role is to keep the population diversity; $\mathbf{p}_{i,k}$ is defined as the best location found by particle i up to time k; and $\mathbf{g}_k$ is the best global position found among all particles in the swarm, up to time k;

Step 3. Each particle position is updated as

$$\mathbf{x}_{i,k+1} = \mathbf{x}_{i,k} + \mathbf{v}_{i,k+1}; \tag{19}$$

Step 4. Check of stopping criteria.

The fitness for the locations $\mathbf{p}_{i,k}$ and $\mathbf{g}_k$ is defined as $f_{i,\text{best}}$ and $f_{g,\text{best}}$, respectively. The objective is then to minimize the difference between $f_{i,\text{best}}$ and $f_{g,\text{best}}$ such that no further improvement is introduced. It normally takes from few hundred to few thousand iterations until convergence is achieved. In the present problem, the values for the positions of the particles $(x_{i,k})$ are the cross-section areas (A_i) at the iteration k.

3.1.2. The genetic algorithm method

The Genetic Algorithms (GA) were developed firmly based on the evolution laws of biological species [4]. These algorithms have their philosophical basis in Darwin's theory on the survival of the best adapted to the environment by means of natural selection. The main steps for a GA applied to damage identification are given below.

Step 1. To generate an initial population with 0's and 1's, for a specified length of the binary chain;
Step 2. To evaluate the adaptability function for each individual;
Step 3. To verify if some stoping criterion is satisfied. If not, go to the next step. If yes, stop the algorithm;
Step 4. To select individuals for cross-breeding;
Step 5. To create a new generation through the cross-breeding and mutation;
Step 6. To evaluate the adaptability of the new generation;
Step 7. To substitute the old generation by the new one. Go to Step 3.

In Step 1, a small population does not has enough genetic material and consequently it does not represent well the project space, while a too big population increases the computational effort, loosing efficiency [4]. In Step 3 the stop criteria is established and the maximum number of generations is set. The computing time or the number of generations without a slight improvement can be used as a parameter for the stop criteria. In Step 4, the simpler idea is to take the probability proportional to the value of the parameter to be found. The better adapted individual will have the bigger probability of choice.

3.1.3. The Luus-Jaakola method

The Luus-Jaakola method (LJ) is a global optimization algorithm aiming at minimizing a given functional E. It was first proposed in 1973 by Luus and Jaakola [9] to solve a non-linear optimization problem, based on a stochastic search.

The technique is conceptually and practically simple, and encompass the following steps.

Step 1. The counter variable k is set to 1. For each counter value and for a specified number of iterations, one has

$$\mathbf{A} = \mathbf{A}^* + \mathbf{Z}\mathbf{d}, \tag{20}$$

where $\mathbf{A}^*$ is the vector of current parameters, $\mathbf{Z}$ is a diagonal matrix containing random numbers in the interval $[-1, 1]$, and $\mathbf{d}$ is a vector containing the diameter of the search regions for the parameters to be identified. At the end of each iteration, $\mathbf{A}^*$ is substituted by $\mathbf{A}$ in the case that this one presents a better configuration, that is, it yields a lower functional value;

Step 2. The search region is contracted according to

$$\mathbf{d}^{k+1} = (1 - \varepsilon)\mathbf{d}^k, \tag{21}$$

where ϵ is an arbitrarily small number. If some stop criterium is reached or the counter is greater than a specified value, the algorithm stops. If not, the algorithm goes back to Step 1.

As aforementioned, the technique is simple and, as it will be shown in the numerical results, it yielded to very fast and surprisingly good results when it was combined with the Sequential Algebraic Algorithm for the damage identification.

3.1.4. The particle collision algorithm method

The Particle Collision Algorithm (PCA) is a relatively new stochastic optimization technique. It was originally developed in the nuclear research area and it was inspired in nuclear particles scattering [19].

In a concise form, the main steps of the canonic PCA method follow.

Step 1. The counter variable is set to 1;

Step 2. An initial solution is randomly generated in the search domain. Let us call it OC (Old Configuration). It is defined by

$$OC = L + (U - L)r, \tag{22}$$

where L and U are, respectively, the lower and upper limits in the search domain, and r is a random number in the interval $[0, 1]$;

Step 3. If the counter achieves the established maximum value, the algorithm stops. If not, a new solution NC (New Configuration) is computed using the relationships

$$\begin{aligned} &NC = OC[(U - OC)r - (OC - L)(1 - r)], \\ \text{where} \quad &NC = L, \quad \text{if} \quad NC < L, \\ \text{and} \quad &NC = U, \quad \text{if} \quad NC > U. \end{aligned} \tag{23}$$

If the value of the objective function for NC is lower than that for OC, then NC takes the value of OC and the algorithm goes to Step 4. Otherwise, goes to Step 5. It is suggested [19] to maintain a variable BC (BestConfig) that retains the value of the best solution at the moment (iteraction). When a new solution is assigned to OC, it must be verified if this solution is better than BC and, if this is the case, make $BC = OC$;

Step 4. The following procedure is then repeated $n_{\exp}$ (number of exploitations) times:

$$\begin{aligned} &NC = OC + [(U^* - OC)r - (OC - L^*)(1 - r)] \\ \text{where} \quad &NC = L, \quad \text{if} \quad NC < L, \\ \text{and} \quad &NC = U, \quad \text{if} \quad NC > U. \end{aligned} \tag{24}$$

In the previous procedure, it is taken

$$\begin{aligned} &U^* = (1 + 0.2r)OC, \\ \text{and } &L^* = (1 - 0.2r)OC. \end{aligned} \tag{25}$$

In this step, therefore, if the objective function for NC is lower than the one for OC then $OC = NC$. When a new solution is adopted as OC it should be checked if it is better than BC and, this being the case, make $BC = OC$. Once the procedure described in this step is repeated $n_{\exp}$ times, go to Step 3;

Step 5. Compute the scattering probability, p_{scat}, as

$$p_{\text{scat}} = 1 - \frac{E(BC)}{E(NC)}. \tag{26}$$

Generate a random number r. If $p_{\text{scat}} > r$ then $OC = NC$ and go to Step 4. Otherwise, go to Step 2.
It is worth noting that by the end of the PCA method the estimate of the objective function minimum is given by BC.

3.1.5. The simulated annealing method

The Simulated Annealing method is based on principles of statistical mechanics. The application of this method in distinct inverse problem areas has proven to succeed [20]. Consider the metal fusion process, where the probability of existence of a certain configuration after a change of energy ΔE in a given equilibrium temperature T is given as

$$p(\Delta E) = \exp\left(\frac{-\Delta E}{k_B T}\right), \tag{27}$$

where k_B is the Boltzmann constant.

A finite number of randomic variations in temperature T is considered. They constitute a cycle in the optimization procedure. The temperature is then reduced according with a cooling pattern, up to a final prescribed value, with the simple procedure

$$T^{n+1} = R_t T^n, \tag{28}$$

where n is an integer and R_t is the cooling rate that establishes the desired annealing scheme. At each new discrete temperature the procedure is repeated.

Let us use the following notation for the variables: $\mathbf{A}$ is the solution in the current iteration; $\mathbf{A}^*$ is the best solution found; E is the objective function; T_0 is the initial temperature; T is the current temperature; and r is a random number in the interval $[0, 1]$. The SA method can then be (briefly) expressed by a 12 steps procedure as follow.

Step 1. Give to $\mathbf{A}$ an initial solution;
Step 2. Make $\mathbf{A}^* = \mathbf{A}$;
Step 3. Define an initial temperature T_0;
Step 4. Verify if the stopping criteria were reached. If yes, stop the procedure;
Step 5. Choose a solution $\mathbf{A}'$ in a neighborhood of $\mathbf{A}$ as

$$\mathbf{A}' = \mathbf{A} + \mathbf{Z}\mathbf{v} \tag{29}$$

where $\mathbf{Z}$ is a diagonal matrix containing random numbers in the interval $[-1, 1]$, and $\mathbf{v}$ is the step vector;
Step 6. Compute $\Delta E = E(\mathbf{A}') - E(\mathbf{A})$;
Step 7. Verify if $\Delta E < 0$;
Step 8. If the comparison stated in Step 7 is *true*, make $\mathbf{A} = \mathbf{A}'$;
Step 9. Otherwise, generate a new random number r' in the interval $[0, 1]$. If $r' < e^{-\Delta E/T}$, then make $\mathbf{A}' = \mathbf{A}$;
Step 10. Go back to Step 5;
Step 11. Update the variable T;
Step 12. Go back to Step 4.

It is worth noting that in Step 9, if $E(\mathbf{A}')$ is greater than or equal to $E(\mathbf{A})$, the Metropolis criterion [12] decides if the point is accepted or not.

3.2. *The Levenberg-Marquardt method*

Briefly, the deterministic Levenberg-Marquardt (LM) method [11] consists in constructing an iterative procedure, which starts with an initial guess $\mathbf{A}^0$, and, at the $(k+1)$-th iteration, the new estimate is given by

$$\mathbf{A}^{k+1} = \mathbf{A}^k + \Delta\mathbf{A}^k, \quad k = 0, 1, \ldots, \tag{30}$$

with the variation $\Delta\mathbf{A}^k$ being computed from

$$\Delta\mathbf{A}^k = -\left(\left(\mathbf{J}^T\right)^k \mathbf{J}^k + \lambda^k \mathbf{I}\right)^{-1} (\mathbf{J}^T)^k\, \Gamma\, \mathbf{r}^k, \tag{31}$$

where λ is a damping parameter that is adjusted at each iteration, $\mathbf{I}$ is the identity matrix, Γ is a relaxation factor, and the elements of the Jacobian matrix $\mathbf{J}$ are defined as

$$J_{ji} = \frac{\partial G_j}{\partial A_i}, \quad j = 1, 2, \ldots, N, \quad i = 1, 2, \ldots, n. \tag{32}$$

The iterative procedure is continued until some convergence criterion is satisfied, for instance, $|E^k| < \epsilon_1$ or $|E^{k+1} - E^k| < \epsilon_2$, where ϵ_1 and ϵ_2 are arbitrarily small numbers.

Being deterministic and based on the Jacobian matrix that depends on the gradients $\partial G_j/\partial A_i$, the LM method presents a fast convergency but with the risk to stop in a relative minimum. Furthermore, as it is shown in [23], there is a strong dependence on the parameter Γ, which must be adjusted in an *ad hoc* manner for each situation. On the other hand, as mentioned before, if the initial guess is a reasonable one – as the one obtained by the output of a stochastic optimization method –, then the LM is likely to achieve the desired minimum with a few iterations.

The parameters of the optimization methods were adopted as follows.

1. PSO: $m = 25, x_{\min} = 0, x_{\max} = 1, c_1 = c_2 = c_3 = 0.05$;
2. GA: max. number of generations $= 20$, population size $= 4$;
3. LJ: $\varepsilon = 0.05, \mathbf{d} = \mathbf{1}$;
4. PCA: $L = 0, U = 1, n_{\exp} = 10{,}000$;
5. SA: $R_t = 0.75, T_0 = 5, \mathbf{v} = \mathbf{1}$;
6. LM: $\Gamma = 10$, λ was set an initial value of 10 and it was dynamically adjusted according to the objective function value E.

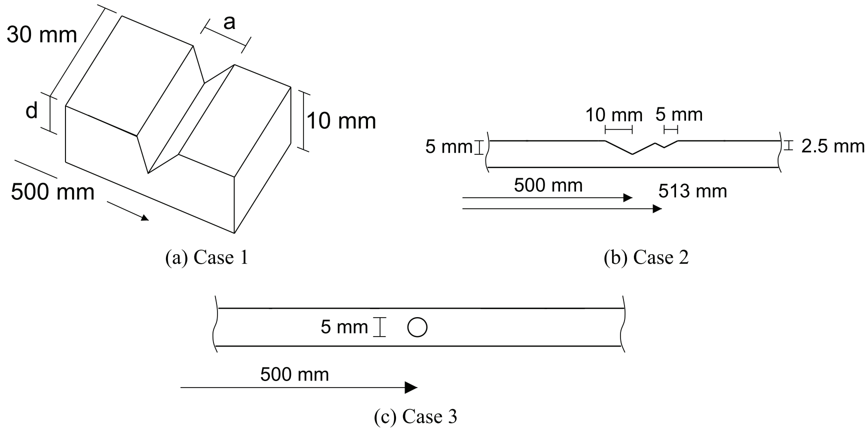


Fig. 1. Damage scenarios imposed to the slender bar.

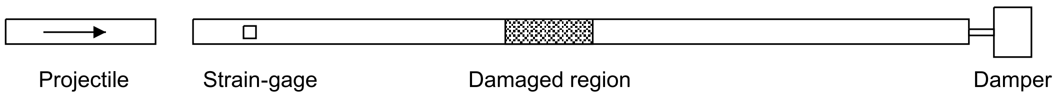


Fig. 2. Depict of an experimental pulse-echo setup.

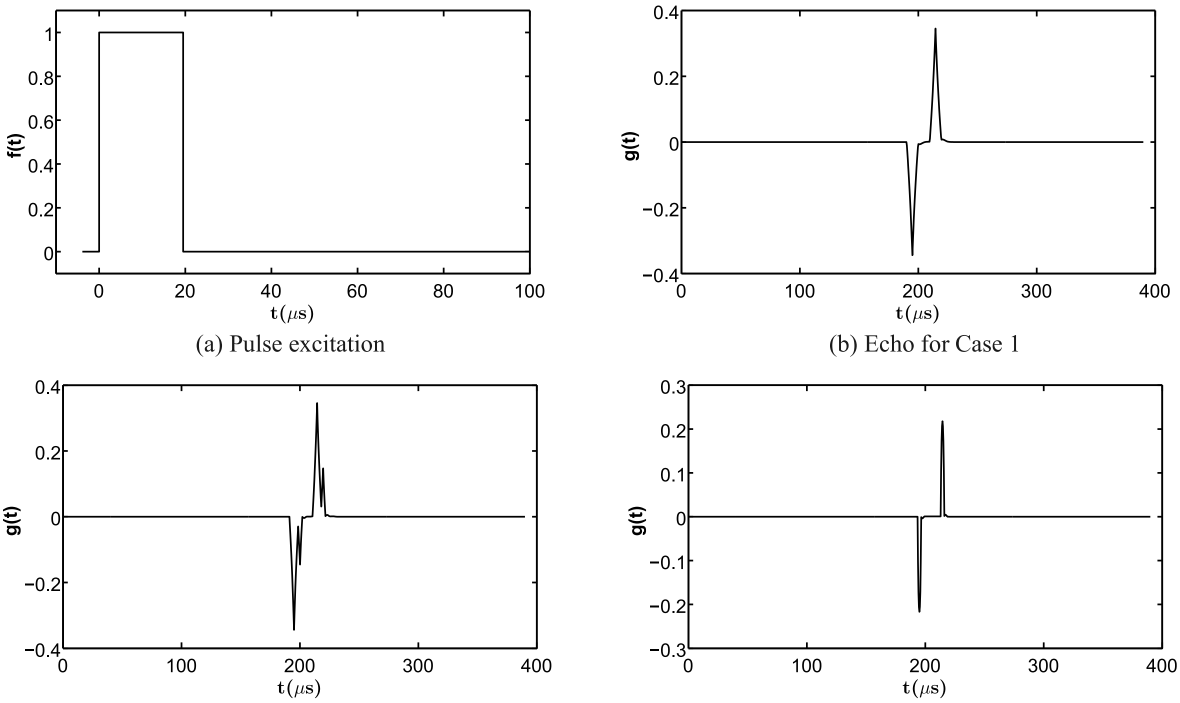
4. Damage scenarios addressed

In order to numerically assess the performance of the proposed damage identification methods, a nonuniform aluminium bar with three distinct damage scenarios is considered in the present work. The structure under concern has length $l = 1000$ mm, nominal thickness $h = 10$ mm, width $w = 30$ mm, yielding a nominal rectangular cross-section area $A = 300$ mm^2, elasticity modulus $E_a = 7.1 \times 10^{10}$ Pa, mass density $\rho = 2.7 \times 10^3$ kg/m^3 and longitudinal plane wave speed $c = 5128$ m/s. As it is well known, such a slender bar works as a plane waveguide, as demonstrated by experimental tests in [15,23]. Therefore, the one-dimensional SAA, presented in Section 2, applies quite well.

Three different damage scenarios, referred to as Cases 1 to 3, as depicted in Fig. 1, are imposed to the structure. Case 1 presents a triangular shape, at the center of the bar, with a maximum depth $d = 5$ mm and maximum length (the triangle base) $a = 30$ mm, as shown in Fig. 1(a). Case 2 presents a double triangular shape, the first one with the same maximum depth and the second one with half of the depth, as shown in Fig. 1(b). Case 3 corresponds to a cylindrical hole crossing the whole bar's width, with a diameter of 5 mm, as shown in Fig. 1(c). Among the damage scenarios, Case 1 was selected as a benchmark for assessing the comparative performance of the considered optimization methods.

For the sake of completeness a brief description of an actual experiment in a non-homogeneous bar is presented. The reader is referred to [23] for more details. Figure 2 illustrates the experimental apparatus, which consists of an improved Hopkinson bar. A strain gage sensor is placed near the left free end of the damaged bar in order to measure both the incident wave pulse (progressive wave) and the corresponding echo (regressive wave), resulted from the interaction of the incident wave with the damage within the bar. A damper at the right end of the bar provides the absorption of the impact energy. The mechanical impact is produced by the collision of a smaller bar (the projectile), with the same diameter, at the left end of the bar under test. This impact produces a roughly rectangular shaped incident progressive wave that travels along the test bar, with a constant speed, crossing the strain gage station and reaching the damaged region.

In the present work, in order to obtain the pulse and echo signals, required for the damage identification process, a pulse-echo synthetic experiment was performed for each damage scenario as follows. A longitudinal rectangular pulse was considered as the excitation signal applied at the left end of the bar. Then, the direct acoustic wave propagation problem is computed by using the SAA and the corresponding *echo* signal, resulted from the iteration between the incident wave *pulse* and the considered damage scenario, is computed.



(a) Pulse excitation (b) Echo for Case 1

(c) Echo for Case 2 (d) Echo for Case 3

Fig. 3. Noiseless pulse and echo signals for Cases 1, 2 and 3.

5. Identification with noiseless data

In this section, the proposed hybrid methods are considered for solving the inverse problem of damage identification in the absence of noise in either the input and echo signals. The rectangular pulse excitation and the corresponding echo signals for Cases 1, 2 and 3 in the absence of noise, are depicted in Fig. 3. These signals were obtained considering an uniform spatial discretization of the bar in layers of length $\Delta x = 1$ mm.

It is worth noting that the echoes themselves provide important and useful information about the damage location. For instance, considering the acoustic wave speed in the present case, $c = 5128$ m/s, and the echo signal for Case 1, Fig. 3(b), which is non-null between approximately 190 and 200 μs, one may infer that the damaged region is located between 487.5 and 512.5 mm, which actually corresponds to the damaged region for Case 1, as depicted in Fig. 1. Therefore, although the bar was discretized into 1000 elements, only 50 parameters were recovered in our examples, due to the restricted dimensions of the damaged regions. However, the damage *shape* cannot be identified without solving the corresponding inverse problem, since it cannot be inferred by inspection from Figs 3(b), (c) or (d).

The accuracy of the results is dependent on the discretization of the bar, that is, of the parameter n. As will be shown in the examples that follow, $n = 1000$ is proved to be a good choice for the considered situations. However, for some actual impedance profiles, as those caused by a corrosion process, probably, to obtain a more precise description of the damage, a greater value of n must be adopted, which corresponds to a greater number of parameters to be recovered within the same damaged region.

Figure 4 shows the results of the identification for, respectively, Cases 1, 2 and 3, by applying each one of the five hybrid methods under concern.

It is worth noting that, since the plane wave model applies to a waveguide, the identification of the *hole* in Case 3 is not expected, but only the corresponding variation of the bar's cross section area. As can be seen in Fig. 4, as expected, for all the three cases the profile recovery is almost perfect for all hybrid optimization techniques. Therefore, one may conclude that, no matter what optimization method is adopted, the proposed damage identification procedure can identify the actual impedance profile when *noiseless data* are under concern.

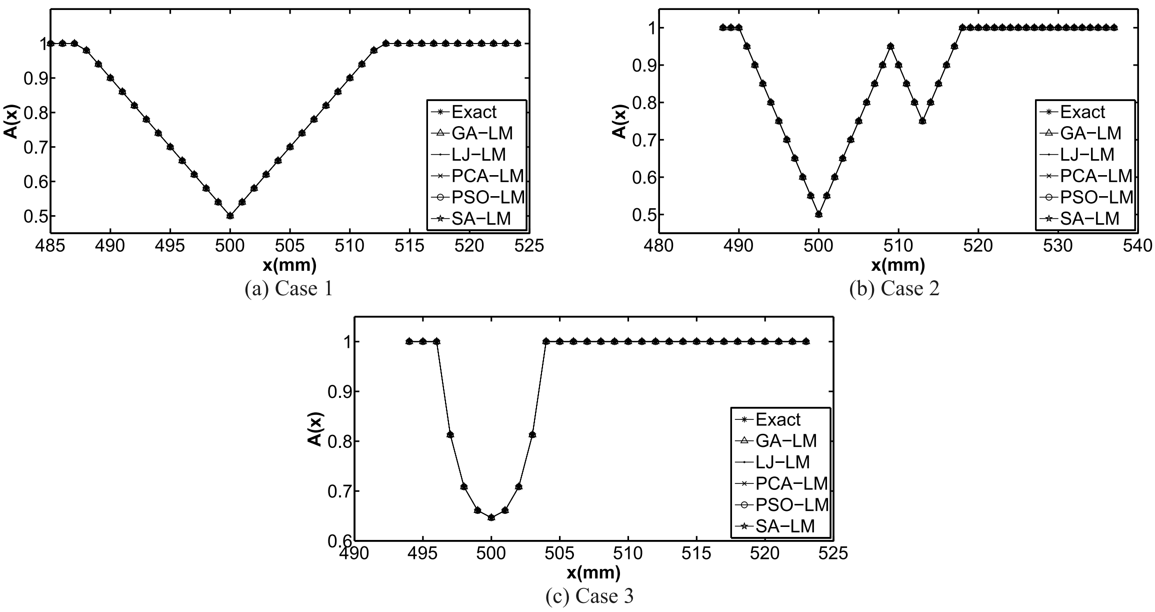


Fig. 4. Identification with noiseless data for Cases 1 to 3.

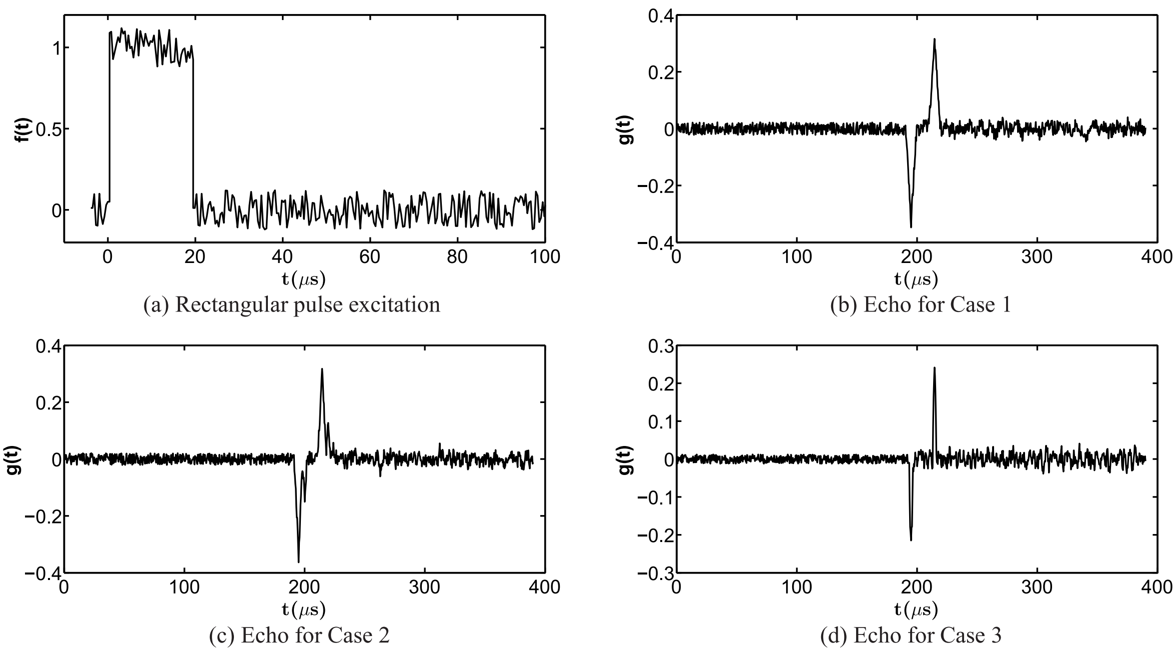


Fig. 5. Pulse and echo signals for Cases 1, 2 and 3 with a $\mathrm{SNR} = 10$ dB.

6. Identification with noisy data

In actual situations, the experimentally obtained pulse and echo signals are always corrupted by some level of noise. Therefore, aiming at analyzing the behavior of the optimization methods in the presence of noisy data, a zero mean random noise was added to both pulse and echo signals. As observed in [23], where an actual pulse-echo experiment in bars is reported, typically a 20 dB of signal to noise ratio (SNR) is found. In the numerical tests considered in the sequel, a higher noise level is adopted, leading to a SNR of 10 dB.

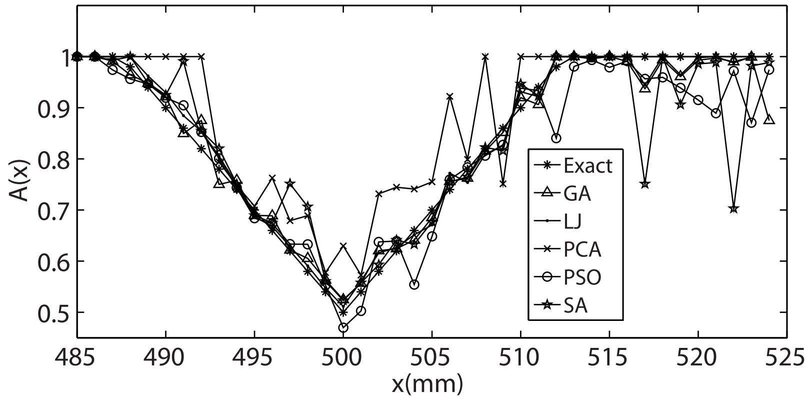
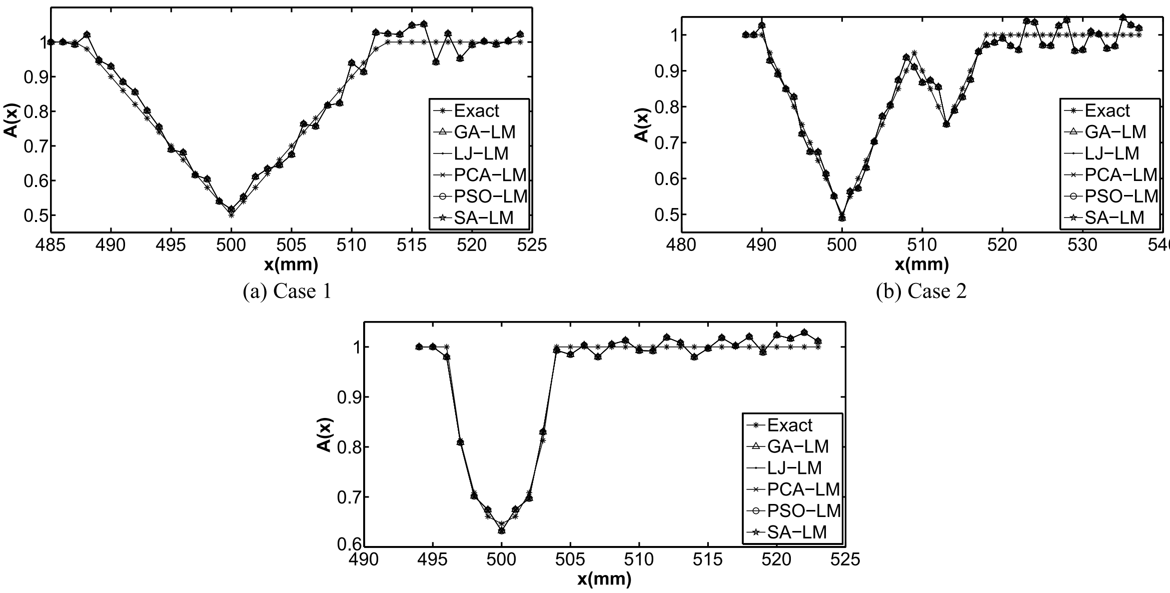


Fig. 6. Damage identification for Case 1: Stochastic methods.



(a) Case 1

(b) Case 2

(c) Case 3

Fig. 7. Damage identification for Cases 1 to 3: Hybrid methods.

Figure 5(a) presents the rectangular pulse excitation with a SNR of 10 dB. Figures 5(b)–(d) present the corresponding echoes for Cases 1, 2 and 3, respectively, with the same level of additive random noise.

In the identification procedure, as aforementioned, each stochastic method is used to generate an initial guess for the Levenberg-Marquardt method. In the present paper, the stopping criterion adopted for the stochastic methods was based on the functional E. That is, if the functional value was below 10^{-2}, the execution of the algorithm is stopped. However, since the five considered stochastic methods are very distinct, the actual final value obtained is different for each method. Figure 6 illustrates the output of the considered stochastic methods in the damage identification for Case 1.

It becomes clear that each stochastic method generates a different solution, as expected. In this case, the PCA technique provided the worst result. However, the output of all stochastic methods represents a good initial guess for the LM one. Indeed, as can be seen from Fig. 7(a), after the complete process, with the five hybrid methods applied, there is no noticeable difference among the identification results.

Figures 7(b) and (c) present the identification results of the hybrid methods for Case 2 and 3, respectively.

In Fig. 7 it can be observed an oscillation of the identified profile, around the exact one for the three cases, and this

Table 1
Performance of the stochastic methods in the benchmark damage identification problem

Method	PSO	LJ	PCA	GA	SA
SNR			Without noise		
Executions – SAA	950	7600	4379	3040	42970
Cost function E	$2.39 \cdot 10^{-2}$	$8.14 \cdot 10^{-4}$	$9.48 \cdot 10^{-2}$	$2.50 \cdot 10^{-3}$	$8.16 \cdot 10^{-5}$
Execution time (s)	21.68	3.15	23.56	23.52	59.28
SNR			10 dB		
Executions – SAA	950	7600	4277	3040	39971
Cost function E	$3.90 \cdot 10^{-2}$	$2.83 \cdot 10^{-3}$	$1.14 \cdot 10^{-1}$	$1.08 \cdot 10^{-2}$	$6.78 \cdot 10^{-4}$
Execution time (s)	22.20	3.15	24.36	27.32	58.60

Table 2
Performance of the Levenberg-Marquardt method alone with an input given by the stochastic methods, in the benchmark damage identification problem

Method	PSO-LM	LJ-LM	PCA-LM	GA-LM	SA-LM
SNR			Without noise		
Iterations – LM	7	7	7	7	8
Cost function E	$1.19 \cdot 10^{-20}$	$1.18 \cdot 10^{-20}$	$1.19 \cdot 10^{-20}$	$1.22 \cdot 10^{-20}$	$1.20 \cdot 10^{-20}$
Execution time (s)	10.07	10.01	10.24	10.09	11.70
SNR			10 dB		
Iterations – LM	9	8	9	8	8
cost function E	$1.99 \cdot 10^{-17}$	$9.88 \cdot 10^{-18}$	$1.99 \cdot 10^{-17}$	$1.99 \cdot 10^{-17}$	$1.99 \cdot 10^{-17}$
Execution time (s)	14.38	12.48	14.41	13.01	12.86

phenomenon is due the high level of noise introduced in the synthetic data. On the other hand, if the identification error is quite the same a performance study – namely, an analysis of the computing time and final error in the cost function – would provide some hints about the methods characteristics and better adequacy *to solve this specific problem*.

7. Performance comparison

This section presents a comparison of the computational efforts of each hybrid method in identifying the damage scenario in Case 1. All algorithms were implemented using the Fortran computational language and the results presented in Tables 1 to 3 were obtained by using a processor Intel Core I5-2410M with 2.30 GHz and 6 GB of RAM memory, running in an operational system Windows 7 of 64 Bits.

Table 1 presents the main figures relative to the first step of the damage identification process, when the stochastic algorithm is executed in the hybrid method. The simulations were performed considering both noiseless data and a SNR of 10 dB applied to the signals. The execution of each algorithm was stopped if the functional value was below 10^{-2}, as before. However, for some techniques, the stopping criterion was imposed in an outer loop of the algorithm and, in that case, a relatively lower functional value may be obtained.

One interesting observation is that the five stochastic methods run with similar computation times for the noiseless and the noisy cases. Among the considered methods, the LJ presented the lower computation time. It is worth noting that the number of executions of the SAA has little influence on the performance of the methods. This is due to the fact that being an exact algebraic solution for the discretized direct problem, the execution of the SAA is extremely fast.

Table 2 presents the comparative performance of the Levenberg-Marquardt method alone with its input given by each one of the stochastic methods, in the benchmark damage identification problem. It can be seen that the computing time is lower than 12 seconds without noise and lower than 15 seconds with SNR of 10 dB. The most important difference among noiseless and noisy data is in the order of the final cost function (10^{-20} and 10^{-17}, respectively).

Table 3 presents the global computing time comparative results among the five considered hybrid techniques, for noiseless and noisy data. It is worth noting that there is no significant influence in the execution time of the level of noise.

Table 3
Comparative performance of the hybrid techniques (computing time) in the benchmark damage identification problem

Method	PSO-LM	LJ-LM	PCA-LM	GA-LM	SA-LM
SNR			Without noise		
Execution time (s)	32.38	13.16	33.80	33.61	70.98
SNR			10 dB		
Execution time (s)	36.58	15.63	38.77	40.33	69.46

As a conclusion, since the obtained final cost function is almost the same and the computing time of the LM method is around the same – from 10.01 to 11.70 seconds without noise and from 12.48 to 14.41 seconds with noise – the performance is governed by the stochastic method.

The Luus-Jakoola/Levenberg-Marquardt hybrid optimization method obtained the lowest execution time, corresponding to 13.16 seconds with noiseless data and 15.63 seconds with SNR of 10 dB as shown in Table 3.

It is important to observe that, if the fastest Hybrid technique run the identification in 15.63 seconds, with noise, the lowest one spent 69.46 seconds. This means that all methods succeed in identifying the damage for our benchmark (Case 1) with a low computational cost. Two main aspects should be stressed here. The first one is that, as aforementioned, the Sequential Algebraic Algorithm, being an exact algebraic solution for the discretized direct problem, runs extraordinarily fast, so that it takes very little time consuming during the several iterations performed by the stochastic methods (see, for instance the third line of Table 1). A second and important feature is that the SAA allows the identification of the parameters in a sequential procedure, and this facilitates to deal with a reasonably great number of parameters to be identified. Indeed, in our benchmark, approximately 50 parameters were recovered.

8. Conclusions

The damage identification in bars built on a longitudinal acoustic wave propagation approach was addressed in the present paper. The direct problem was solved trough the Sequential Algebraic Algorithm (SAA), a reliable and fast technique, as it was proven in a previous paper [23], where the SAA method was able to preview the experimentally obtained echo generated by an inhomogeneity in the structure due to the presence of damage. The corresponding inverse problem of damage identification was then addressed considering five distinct hybrid optimization techniques, based on the combination of one of the five stochastic optimization methods: PSO, GA, LJ, PCA and SA, with the deterministic Levenberg-Marquardt method.

Three damage scenarios were addressed in the numerical tests. Also, two conditions were considered: noiseless data and noisy data with the relatively high signal to noise ratio (SNR) of 10 dB. With the noiseless data, the damage recovery was almost perfect for the three considered damage scenarios and with all five hybrid optimization techniques. The numerical tests with SNR of 10 dB showed good identification results, but, due to the presence of noise in both the pulse and echo signals, the estimated cross-section area profile presented some small oscillations further the actual damaged regions. Therefore, the damage identification procedure built on the SAA was shown robust with respect to noise corrupted signals, yielding to satisfactory results even in the presence of a high level noise.

It is worth noting that in the first step of the hybrid optimization procedure, the identification result strongly depends on the stochastic method considered, as shown in Fig. 7. However, after the second step, in which the LM method is applied, the damage recovery was almost the same for the five hybrid techniques, as shown in Figs 8–10.

In order to assess the computational performance of the methods, one damage scenario (Case 1) was considered as a benchmark. Since none of the techniques presented a significant memory charge, the execution time was the main feature under concern. It became clear that the LJ-LM hybrid technique presented the lowest execution time, even with a larger number of execution of the direct problem than, for instance, the PSO-LM technique, as seen in Table 1.

It is worth emphasizing that the sequential algebraic algorithm (SAA) yields an exact solution for the discretized inhomogeneous wave propagation phenomenon without adding a relevant computational burden. Besides, its formulation enables one to identify only one parameter at once, instead of all the parameters together, as usual. This feature facilitates the use of the stochastic optimization techniques and, of course, the hybrid ones. The number of identified parameters in each considered damage scenario was of the order of 50, which is not usual in optimization procedures. As a consequence, not only the damage location and severity were identified but the damage shape was also recovered.

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References

[1] J.V.A. dos Santos, N.M.M. Maia, C.M.M. Soares and C.A.M. Soares, Structural damage identification: A survey, in: *Trends in Computational Structures Technology, Saxe-Coburg Publications, Stirlingshire*, Chapter 1, B.H.V. Topping and M. Papadrakakis, eds, 2008, pp. 1–24, doi:10.4203/csets.19.1.
[2] J.H. Cantrell and W.T. Yost, Nonlinear ultrasonic characterization of fatigue microstructures, *Int J Fatigue* **23** (2001), 487–480.
[3] R. Gangadharan, D.R. Mahapatra, S. Gopalakrishnan, C.R.L. Murthy and M.R. Bhat, On the sensitivity of elastic waves due to structural damages: Time-frequency based indexing method, *Journal of Sound and Vibration* **320** (2009), 915–941.
[4] D.E. Goldberg, *Genetic Algorithms for Search, Optimization and Machine Learning*, Reading, MA: Addison-Wesley, 1989.
[5] J. Kennedy and R.C. Eberhart, Swarm intelligence, Morgan Kaufmann Publishers, 2001.
[6] S. Kirkpatrick, C.D. Gelatt and M.P. Vecchi, Optimization by simulated annealing, *Sience* **220** (1983), 671–680.
[7] U. Lee, A reduced-domain method of structural damage identification: Application to a spectral element beam model, *Shock and Vibration* **10** (2003), 313–324.
[8] W. Lestari, Application of wave propagation analysis for damage identification in composite laminated beams, *J Composite Materials* **39**(22) (2005), 1967–1984.
[9] R. Luus and T.H.L. Jaakola, Optimization by direct search and systematic reduction of the size of search region, *AIChE Journal* **19** (1973), 760–788.
[10] G. Manson and R.J. Barthorpe, Advanced feature selection for simplified pattern recognition within the damage identification framework, *Shock and Vibration* **17** (2010), 589–599.
[11] D.W. Marquardt, An algorithm for least-squares estimation of nonlinear parameters, *J Soc Industr Appl Math* **11** (1963), 431–441.
[12] N. Metropolis, A. Rosenbluth, M.N. Rosenbluth, H.A. Teller and E. Teller, Equation of state calculation by fast computing machines, *Journal of Chemical Physics* **21** (1953), 1087–1092.
[13] A. Nag, D.R. Mahapatra and S. Gopalakrihnan, Identification of delamination in composite beams using spectral estimation and a genetic algorithm, *Smart Materials and Structures* **11**(6) (2002), 899–908.
[14] P.B. Nagy, Fatigue damage assessment by nonlinear ultrasonic material characterization, *Ultrasonics* **36** (1998), 375–381.
[15] S.H. Nia, J.J. Charngc and K.F. Lod, Low-strain integrity testing of drilled piles with high slenderness ratio, *Computers and Geotechnics* **33** (2006), 283–293.
[16] A.D. Pierce, Acoustics: An introduction to its physical principles and applications, *Acoustical Society of America*, Woodbury, (1989).
[17] E.A. Robinson and S. Treitel, Geophysical signal analysis, Englewood Cliffs, Prentice Hall, 1980.
[18] S. Rucevskis and M. Wesolowski, Identification of damage in a beam structure by using mode shape curvature squares, *Shock and Vibration* **17** (2010), 601–610.
[19] W.F. Sacco, C.R.E. Oliveira and C.M.N.A. Pereira, Two sthocastic optimization algorithms applied to nuclear reactor core design, *Progress in Nuclear Energy* **48** (2006), 525–539.
[20] A.J.S. Neto and F.J.C.P. Soeiro, Estimation of phase function of anisotropic scattering with a combination of gradient based and stochastic global optimization method, *Proceeding of the* 5^{th} *World Congress Computational Mechanics*, Viena, (2002).
[21] R.A. Tenenbaum and M. Zindeluk, An exact solution for the one-dimensional elastic wave equation in layered media, *J Acoust Soc Am* **92**(6) (1992), 3364–3370.
[22] R.A. Tenenbaum, L.T. Stutz and K.M. Fernandes, Comparison of vibration and wave propagation approaches applied to assess damage influence on the behavior of Euler-Bernoulli beams, *Computers and Structures* **89** (2011), 1820–1828.
[23] R.A. Tenenbaum, K.M. Fernandes, L.T. Stutz and A.J.S. Neto, Damage identification in bars with a wave propagation approach and a hybrid optimization method, *Shock and Vibration* **19** (2012), 301–321.
[24] H.F.C. Velho, F.M. Ramos, E.S. Chalhoub, S. Stephany, J.C. Carvalho and F.L. Souza, Inverse problems in space science and technology, *Proceedings of the* 5^{th} *International Conference on Industrial and Applied Mathematics*, Sidney, (2003).
[25] J.R. Wait, G.H. Park and C.R. Farrar, Integrated structural health assessment using piezoelectric active sensors, *Shock and Vibration* **12** (2005), 389–405.
[26] V. Giurgiutiu, A. Zagrai and J. Bao, Damage identification in aging aircraft structures with piezoelectric wafer active sensors, *Journal of Intelligent Material Systems and Structures* **15** (2004), 673–687.
[27] R.P. Dalton, P. Cawley and M.J.S. Lowe, The potential of guided waves for monitoring large areas of metallic aircraft fuselage structures, *Journal of Nondestructive Evaluation* **20** (2001), 29–46.

[28] M. Rucka, Experimental and numerical study on damage detection in an L-joint using guided wave propagation, *Journal of Sound and Vibration* **329** (2010), 1760–1779.
[29] W.M. Ostachowicz, Damage detection of structures using spectral finite element method, *Computers and Structures* **86** (2008), 454–462.
[30] M. Krawczuk, M. Palacz and W. Ostachowicz, The dynamic analysis of a cracked Timoshenko beam by the spectral element method, *Journal of Sound and Vibration* **264** (2003), 1139–1153.
[31] H. Peng, G. Meng and F. Li, Modeling of wave propagation in plate structure susing three-dimensional spectral element method for damage detection, *Journal of Sound and Vibration* **320** (2009), 942–954.