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Reliability based design of driven pile groups using combination of pile driving equations and high strain dynamic pile monitoring

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ABSTRACT

Reliability based design (e.g., load and resistance factor design – LRFD) aims at meeting a maximum permissible probability of failure (target reliability) for engineered systems or major elements thereof. For deep foundations, such as driven pile groups, statistical parameters of the random load and target reliabilities are naturally defined for entire pile groups, while pile driving criteria for stopping pile advance are required for individual piles. We present an approach using dynamic equations (e.g., Gates) and dynamic monitoring (e.g., PDA/CAPWAP) for estimating axial pile resistances. Dynamic equations are site-specifically calibrated to dynamic monitoring results from test pile programs, for example, and resistance estimates of production piles from equations (available at all piles) and monitoring (only available at monitored piles) are combined by best linear unbiased estimation (BLUE). Resulting resistance estimates and uncertainties of all piles in a group are further combined to obtain LRFD resistance factors Φ for pile groups as well as explicit pile driving criteria for individual piles. An iteration procedure is presented to account for the possible presence of previously driven piles in a group. A practical example and charts of Φ as a function of the degree of monitoring (percentage of piles monitored in a group) are used to demonstrate and discuss results.

1. Introduction

Within the category of deep foundations, driven piles are a common alternative to drilled shafts. Depending on the stage of implementation, numerous methods exist for the design and quality control of driven piles. These methods may be grouped into three major categories [1]: (1) "Static equations" (e.g., Tomlinson or Schmertmann methods) using measurements of certain soil parameters, SPT and/or CPT data. (2) "Dynamic equations" (e.g., ENR or Gates equations) using information from the pile driving process, such as blow count and hammer properties. (3) "Dynamic measurements" (e.g., Case method or Case Pile Wave Analysis Program – CAPWAP) using data from pile driving analyzer (PDA), i.e., high strain dynamic testing (HSDT). More recently, embedded data collectors (EDC) are available as an alternative to PDA with gages at the bottom as well as at the top of the pile, and results compare well to PDA/CAPWAP [2] as well as static load tests [3]. In a typical driven pile project (1) is applied in the pre-construction phase for pile design, while (2) and (3) are used as pile driving criteria during construction for stopping pile advance and may be adjusted according to static validation load tests on a number of piles [4,5].

Reliability based design (e.g., load and resistance factor design – LRFD; [1,6]) aims at achieving a prescribed level of reliability (maximum permissible probability of failure) for engineered systems. This requires a conceptual model, which is consistent with the design and construction procedure (e.g., the pile driving criteria) and accounts for all types of uncertainties involved (e.g., uncertainties of prediction methods, soil parameters or due to spatial variability of ground properties within a site). Many approaches make use of pile load test databases to determine reliabilities of existing foundations or to establish LRFD resistance factors for future design purposes. Under axial loads, these approaches include Allen [7] for dynamic pile driving equations as well as Kwak et al. [8] for design based on SPT data. McVay et al. [9], Paikowsky [1] and Yoon et al. [10] investigate reliability based performance of a series of different static and dynamic methods, while Zhang [5], Zhang et al. [11] and Liang and Yang [12] propose the use of dynamic measurements and static load tests for reliability based quality control at the end of the construction phase. In an effort to reduce design uncertainty based on large and possibly heterogeneous load test databases, Zhang et al. [13] apply Bayesian updating to incorporate less variable regional and site-specific information. Alternative ap-

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Dimensionless

$CV_{g(pre)}$	estimation uncertainty of R_g
CV_{g0}	estimation uncertainty of R_g for $\rho_s = 0$
CV_{g1}	estimation uncertainty of R_g for $\rho_s = 1$
CV_m	estimation uncertainty of $\lambda_m R_m$; coefficient of variation of ε_m
CV_p	estimation uncertainty of $\lambda_p R_p$; coefficient of variation of ε_p
CV_{pm}	estimation uncertainty of R^*_{pm}
CV_Q	coefficient of variation of random load
CV_{QD}	LRFD dead load coefficient of variation
CV_{QL}	LRFD live load coefficient of variation
CV_{Rm}	coefficient of variation of observed values of R_m at a site
CV_{Rp}	coefficient of variation of observed values of R_p (or R_p') at a site
CV_ε	coefficient of variation of estimation error
E_{mp}	auxiliary variable defined as $E[R_m/R_p]E[R_p]/E[R_m]$
E_{pm}	auxiliary variable defined as $E[R_p/R_m]E[R_m]/E[R_p]$
i, j	index variables
n_p	number of piles in a group
n_m	number of monitored piles in a group
n_0	number of previously driven piles in a group
w_m	BLUE weight of R^*_m
w_p	BLUE weight of R^*_p
β	LRFD reliability index
γ_D	LRFD dead load factor
γ_L	LRFD live load factor
$\varepsilon_{i,j}$	estimation errors for different piles in a group
ε_m	random estimation error of R_m with respect to R_t
ε_p	random estimation error of R_p with respect to R_t
λ_m	bias of R_m with respect to R_t
λ_p	bias of R_p with respect to R_t
λ_R	LRFD resistance bias factor
λ_{QD}	LRFD dead load bias factor
λ_{QL}	LRFD live load bias factor
ρ_{pm}	correlation coefficient between estimation errors of R_p (or R_p') and R_m
ρ_s	average correlation coefficient of estimation errors between different piles in a group
Φ	LRFD resistance factor

Forces

Q_{des}	LRFD nominal design load (expectation of random load)
Q_D	LRFD dead load
Q_L	LRFD live load
R_t	true pile resistance
$R_{m(i)}$	estimated pile resistance from dynamic measurements (of i-th pile in group)
$R_{p(i)}$	estimated pile resistance from dynamic equation (of i-th pile in group)
$R_{p(i)}$'	$R_{p(i)}$ after site-specific adjustment to R_m
R^*_m	R_m after bias correction
$R^*_{p(i)}$	$R_{p(i)}$ or $R_{p(i)}$' after bias correction
$R^*_{pm(i)}$	BLUE estimate of pile resistance based on R^*_p and R^*_m
R_g	estimated (nominal) resistance of pile group
R_{gt}	true (load tested) resistance of pile group
R_n	estimated (nominal) resistance of single pile (equal to expectation of R_t)
R_{n0}	uniform nominal resistance of each previously installed piles in a group

Others

a, b	linear regression coefficients
A, B, C, μ	auxiliary variables in dimension of force^2
$E[]$	expectation operator
H	hammer stroke in meters
N	blow count in blows per meter
W	hammer weight in kilograms

proaches combining uncertainties of relevant design parameters by propagation through design equations rather than comparing predictions against load test results are taken by Foye et al. [14] and Kim et al. [15]. These methods do not rely on the compilation of pertinent load test databases, however, they may not fully account for uncertainties of the design equations (method error) and statistical properties of some design parameters (including correlations between parameters) may not always be well defined.

While the vast majority of reliability studies focus on individual piles (or even side and tip resistances separately), it is logical that target reliabilities should apply to entire systems, i.e., containing the superstructures [16]. However, due to the generally large complexity of the problems (e.g., an entire bridge with many elements influencing each other to a smaller or larger degree), super and substructure design are widely decoupled in the sense that geotechnical engineers receive prescribed target reliabilities and load properties for each foundation element (e.g., one driven pile group). It has been recognized that reliabilities at the pile and group level may be drastically different due to effects of a rigid pile cap and pile redundancy within a group (i.e., failure of a single pile does not necessarily lead to failure of the whole group). In order to account for the latter, it has been suggested and become part of design standards that assigned pile reliabilities should be smaller than required target reliabilities of pile groups [1,9,16]. In the present work, we establish a methodology, which aims to achieve a given target reliability at the pile group level by defining pile driving criteria for individual piles based on the number of piles in a group and a combination of pile resistance predictions from dynamic equations and an arbitrary number of dynamic measurements. We hereby consider the predictions from dynamic measurements as reliably calibrated to static load test databases (e.g., [1]) and perform site-specific calibration of dynamic equations with respect to dynamic measurements. This is to reduce the influence of variable driving equipment performance between sites [4]. While the following sections deal with single piles and the calibration and combination of predictions from dynamic equations and measurements, subsequent sections focus on the pile group level and the development of LFRD resistance factors. A practical example problem and charts are presented thereafter to illustrate the findings.

2. Theoretical development

2.1. Site specific calibration of prediction equation

For the purpose of illustrating the statistical concept, assume a large site with many statically load tested driven piles, such that all "true" pile resistances R_t are random, but known. Note that herein, pile resistance always refers to the total axial resistance in general, which is the sum of side plus tip contributions. For every pile it is generally possible to make a prediction of R_t by one of the available dynamic equations. This will be called the "predicted value" R_p in the sequel and it is a random variable with bias and uncertainty with respect to R_t. For example, if the Federal Highway Administration (FHWA) modified Gates equation [1] is used, one obtains after conversion to SI units

$$R_p = 0.021\sqrt{WH}\log_{10}(0.254N) - 0.445 \qquad (1)$$

where R_p is the Davisson resistance in MN, W is the hammer weight in kilograms, H the hammer stroke in meters and N the blow count in blows per meter. We further define a "monitored value" R_m of R_t, which is also random and obtained from possible dynamic measurements (e.g., PDA or EDC) during pile driving. Through a comprehensive database analysis of load tests and using the Davisson criterion, R_t/R_m from using PDA with CAPWAP analysis, for example, has been found to have an expectation (LRFD "bias") of λ_m = 1.16 with a coefficient of variation CV_m = 0.34 [1].

Analogous values of bias λ_p and uncertainty CV_p for R_p are also available; however, variability in hammer performance between and even during particular driving jobs may be significant [4]. In order to control this variability at least between different sites without significant increase in cost due to static load testing, λ_m and CV_m are assumed to be reliably known and we propose to calibrate R_p indirectly to R_m rather than directly to R_t. Respective data are typically available as depth profiles of R_p and R_m from preliminary test pile programs at a site. According to LRFD practice the relationships

$$R_t = \lambda_p R_p \varepsilon_p \tag{2}$$

$$R_t = \lambda_m R_m \varepsilon_m \tag{3}$$

apply, where ε_p and ε_m are random error terms (independent of R_p and R_m, respectively) of unit expectation and coefficients of variation CV_p and CV_m. Since both estimators are unbiased, the expectations of the right-hand-sides of Eqs. (2) and (3) have to be equal to the expectation of R_t, such that

$$\lambda_p = \frac{\lambda_m E[R_m]}{E[R_p]} \tag{4}$$

may be obtained with $E[\,]$ denoting the expectation operator. Knowing λ_m from historical data and $E[R_m]$ and $E[R_p]$ from the sample means of observed data, λ_p may be inferred. Furthermore, the coefficients of variation of Eqs. (2) and (3) may be written as $CV_{Rp}^2 + CV_p^2$ and $CV_{Rm}^2 + CV_m^2$, respectively, where CV_{Rp} and CV_{Rm} are the coefficients of variation of observed values of R_p and R_m. Both Eqs. (2) and (3) have to reproduce the coefficient of variation of R_t leading to

$$CV_p^2 = CV_{Rm}^2 + CV_m^2 - CV_{Rp}^2 \tag{5}$$

with CV_m again known from historical data. In analogy to standard regression theory, CV_{Rp}^2 (or CV_{Rm}^2) may be regarded as the portion of variability in R_t explained by the estimator, while CV_p^2 (or CV_m^2) represents the complementary portion not explained by the estimator.

The bias corrected estimators $\lambda_p R_p$ and $\lambda_m R_m$ are expected to be correlated to each other as their common purpose is to closely reproduce variable values of R_t. The respective relative estimation errors ε_p and ε_m about any fixed R_t may also possess a non-zero correlation coefficient ρ_{pm} to be determined. Eliminating R_t from Eqs. (2) and (3) gives $\lambda_m R_m/(\lambda_p R_p) = \varepsilon_p/\varepsilon_m$ which possesses an expectation E_{mp} equal to $E[R_m/R_p]E[R_p]/E[R_m] = E[\varepsilon_p]/E[\varepsilon_m](1 + CV_m^2 - CV_m CV_p \rho_{pm})$. The left-hand-side is found using Eq. (4) and may be evaluated from field observations of R_p and R_m. The right-hand-side is derived by Journel and Huijbregts [17, p. 426] using a low order approximation. For log-normal ε_p and ε_m in combination with ρ_{pm} = 0 it can be shown (using [18, p. 103 and 645]) that this approximation becomes exact. In the same way, an expectation E_{pm} for the reciprocal may be expressed as $E[R_p/R_m]E[R_m]/E[R_p] = E[\varepsilon_m]/E[\varepsilon_p](1 + CV_p^2 - CV_m CV_p \rho_{pm})$. With this, the average (E_{mp} + E_{pm})/2, for example, may be obtained from observations and used to infer ρ_{pm} as

$$\rho_{pm} = \frac{1 + 0.5(CV_m^2 + CV_p^2 - E_{mp} - E_{pm})}{CV_m CV_p} \tag{6}$$

where it is recalled that $E[\varepsilon_p]$ = $E[\varepsilon_m]$ = 1. In summary, in the above development it is assumed that λ_m and CV_m are reliably known from databases. This immediately implies that the expectation and coefficient of variation of R_t are also known reliably from Eq. (3), if sufficient observations of R_m are available. Based on this knowledge Eqs. (4) and (5) allow inferring λ_p and CV_p from a series of observed values of R_m and R_p at a site (e.g., from depth profiles of preliminary test piles). Similarly, Eq. (6) explores the observed expectations E_{mp} and E_{pm} to infer ρ_{pm}; the use of (E_{mp} + E_{pm})/2 as opposed to E_{mp} or E_{pm} alone is hereby thought to avoid ambiguity (possibly different values of ρ_{pm} based on E_{mp} or E_{pm} alone). Most importantly, neither of Eqs. (4)–(6) requires values of true load test resistances R_t.

It is noted, however, that the bias model of Eqs. (2) and (3) is purely proportional, i.e, it is based on the assumption that scatterplots of R_t versus R_p and R_t versus R_m form clouds around a straight line through the origin. As a consequence, data points of R_m versus R_p should also scatter around a straight line through the origin. If preliminary data indicates that this is not the case at a given site, then the prediction equation for R_p (e.g., Eq. (1)) should be revised to achieve this proportionality. A modified estimator R_p' = a + bR_p based on standard linear regression between R_p and R_m can be used in Eqs. (4)–(6) instead of R_p (see practical example in Section 3). The adjustment by linear regression then implies that $E[R_p']/E[R_m]$ = 1 and CV_{Rp} < CV_{Rm}, which simplifies Eqs. (4) and (6) and assures CV_p > CV_m in Eq. (5).

2.2. Combining predicted and monitored resistances of a single pile

Using the two unbiased estimators $R_{p*} = \lambda_p R_p$, $R_{m*} = \lambda_m R_m$ and of R_t with their respective uncertainties, a best linear unbiased estimator (BLUE; [19, p. 278]) of the form

$$R_{pm}^* = w_p R_{p*} + w_m R_{m*} \tag{7}$$

is used with weights w_p and w_m to obtain an optimal (in the sense of unbiasedness and minimum error variance) estimate R_{pm}^* of R_t. In spatial interpolation this is equivalent to solving the Ordinary Kriging system, which is here formulated based on estimation error variances,

$$\begin{bmatrix} E[R_t]^2 CV_p^2 & E[R_t]^2 CV_p CV_m \rho_{pm} & 1 \\ E[R_t]^2 CV_p CV_m \rho_{pm} & E[R_t]^2 CV_m^2 & 1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} w_p \\ w_m \\ \mu \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tag{8}$$

and where $E[R_t]$ is used as the common expectation of R_{p*} and R_{m*}. Thus, the products on the diagonal of the coefficient matrix represent the variances of estimation errors ε_p and ε_m, and the off-diagonal product represents the respective covariance. Parameter μ is a Lagrangian operator, which is not further required in the sequel, and the zeros on the right-hand-side are due to independence between estimation errors and true resistances. Solving Eq. (8) gives

$$w_p = \frac{CV_m^2 - CV_m CV_p \rho_{pm}}{CV_p^2 + CV_m^2 - 2CV_m CV_p \rho_{pm}} \tag{9}$$

$$w_m = \frac{CV_p^2 - CV_m CV_p \rho_{pm}}{CV_p^2 + CV_m^2 - 2CV_m CV_p \rho_{pm}} \tag{10}$$

with an estimation uncertainty $CV_{pm}^2 = w_p^2 CV_p^2 + w_m^2 CV_m^2 + 2w_p w_m CV_p CV_m \rho_{pm}$ of R_{pm}^* as

$$CV_{pm}^2 = \frac{CV_p^2 CV_m^2 (1 - \rho_{pm}^2)}{CV_p^2 + CV_m^2 - 2CV_m CV_p \rho_{pm}} \tag{11}$$

It can be shown that CV_{pm}^2 is always less than or equal to both CV_p^2 and CV_m^2, which illustrates the benefit of using a combination of two estimates for R_t. This benefit increases as the two estimates become more independent (ρ_{pm} approaching zero). It is further noted that Eqs. (9)–(11) for ρ_{pm} = 0, and when expressed in variances rather than CV's, are formally identical to results of a Bayesian updating approach by

Zhang et al. [13] between global and regional data. Finally, for unmonitored piles, i.e., when R_m is not available, CV_m/CV_p may be set to a very large value leading to the trivial results of w_p = 1, w_m = 0 and $CV_{pm} = CV_p$. In cases when a number of piles is monitored, but ρ_{pm} is not reliably known or close to one, it is a conservative approximation to disregard predicted resistances at monitored piles, i.e., to use $CV_p/CV_m >> 1$ leading to w_p = 0, w_m = 1 and $CV_{pm} = CV_m$.

2.3. Pile groups with no previously installed piles

Consider a pile group of n_p piles, all of which are still to be driven and where an arbitrary number n_m of which are to be monitored. Using R_{p*} and R^*_{pm} to estimate resistances of unmonitored and monitored piles in the group, respectively, an unbiased estimate of total group resistance R_g is obtained as the sum of each pile's resistance estimate (considering a group efficiency factor of one; [16]). Due to unbiasedness, R_g represents the expectation of true pile group resistance at a particular location and it is, hence, equivalent to the associated "nominal" pile group resistance in LRFD context.

$$R_g = \sum_{i=1}^{n_m} R^*_{pmi} + \sum_{i=n_m+1}^{n_p} R^*_{pi} = n_p R_n \quad (12)$$

This assumes no contribution of the pile cap and that pile spacing is large enough (e.g., over two times pile diameter), such that the possibility of block failure may be excluded [16]. It is further reasonable to assume that all piles are driven to the same nominal resistance R_n, such that the final expression of Eq. (12) is valid. Hereby, and in analogy to R_g, R_n is the unbiased estimate (expectation) of true pile resistance given by R_{p*} and R^*_{pm} for unmonitored and monitored piles, respectively. The next goal is to find the value of R_n, such that driving criteria for individual piles (monitored and unmonitored) may be established, while satisfying the target reliability at the group level.

For combining pile resistance uncertainties into a group resistance uncertainty it is necessary to consider possible correlation of estimation errors between different piles in a group. Zero or small correlation may be expected if estimation errors are mainly caused by random factors that are independent from pile to pile (e.g., instrumentation/operation errors, rapidly changing ground conditions, etc.). Stronger correlations will occur if estimation errors are influenced by site or driving conditions, which vary between pile groups, but are relatively constant for a single pile group. Hence, in general, estimation error may be regarded as a spatially random function possessing a spatial covariance function (or variogram) with a particular correlation length [17–19]. However, it is unlikely to obtain sufficient load test data at a site for calculating estimation errors and their full variogram for exact evaluation of pile group uncertainty. As an alternative, an average spatial correlation coefficient ρ_s is introduced, which expresses the average degree of correlation between estimation errors of different piles in a group. Similar to Eq. (6), ρ_s may be determined from the relationship $E[\varepsilon_i/\varepsilon_j] = E[\varepsilon_i]/E[\varepsilon_j](1 + CV^2_{\varepsilon j} - CV_{\varepsilon i}CV_{\varepsilon j}\rho_s)$, where ε_i and ε_j are known estimation errors (from Eq. (2) or (3)) at two different piles, which are not separated further than the maximum pile separation distance in a production pile group. In other words, $E[\varepsilon_i/\varepsilon_j]$ may be evaluated by pairing up all possible combinations of load tested piles, eliminating those pairs that are separated by more than the maximum pile distance in a group and computing the empirical mean of all remaining ratios $\varepsilon_i/\varepsilon_j$. Note hereby, that piles paired up with themselves (zero separation distance) are not included and that each valid pile pair contributes two ratios $\varepsilon_i/\varepsilon_j$ and $\varepsilon_j/\varepsilon_i$ to the mean (which assures $E[\varepsilon_i/\varepsilon_j] > 1$). Knowing also that $E[\varepsilon_i] = E[\varepsilon_j]$ and $CV_{\varepsilon i} = CV_{\varepsilon j}$ it is found that

$$\rho_s = 1 - \frac{E[\varepsilon_i/\varepsilon_j] - 1}{CV^2_\varepsilon} \quad (13)$$

Note that this method of estimating ρ_s is analogous to using a single point of the experimental variogram, except for exploring $E[\varepsilon_i/\varepsilon_j]$ instead of $E[(\varepsilon_i - \varepsilon_j)^2]$. $E[\varepsilon_i/\varepsilon_j]$ and Eq. (13) may be applied to both predicted ($CV_\varepsilon = CV_p$) and monitored ($CV_\varepsilon = CV_m$) resistance errors. However, practical applicability may still be limited by requiring a sufficient number of load tests (e.g., >20 load test pairs equivalent to 7 load tests in one group) at short separation distances. As an alternative, based on the definition of ρ_s given here, it will be derived further below in terms of load test results on entire pile groups (rather than separate nearby single piles). For this purpose, the resistance estimation uncertainty of pile groups is required.

The variance of a sum is known to be equal to the sum of all elements in the variance–covariance matrix of the summands. Applying this to R_g from Eq. (12) we obtain a symmetric n_p by n_p matrix and a pile group uncertainty CV^2_g after division by R^2_g of

$$\begin{aligned} CV^2_g &= \tfrac{1}{n_p^2}(A + B + C) \\ A &= n_m CV^2_{pm} + (n_p - n_m)CV^2_p \\ B &= n_m(n_m - 1)CV^2_{pm}\rho_s + (n_p - n_m)(n_p - n_m - 1)CV^2_p\rho_s \\ C &= 2n_m(n_p - n_m)(w_pCV^2_p + w_mCV_pCV_m\rho_{pm})\rho_s \end{aligned} \quad (14)$$

The two terms in A correspond to the sums of the variances of n_m monitored and $n_p - n_m$ unmonitored piles. The first and second terms in B are the sums of all covariances between monitored and unmonitored piles, respectively. Finally, C represents the sum of all covariances between monitored and unmonitored piles. Eq. (14) implies that ρ_s applies to predicted and monitored estimation errors and that the correlation between a prediction error at one pile location and a monitored resistance error at another pile location is equal to $\rho_{pm}\rho_s$. Denoting by CV_{g0} and CV_{g1} the uncertainties for the limiting cases of ρ_s = 0 and ρ_s = 1 Eq. (14) leads to

$$CV^2_{g0} = \frac{1}{n_p}\left[\frac{n_m}{n_p}CV^2_{pm} + \left(1 - \frac{n_m}{n_p}\right)CV^2_p\right] \quad (15)$$

$$\begin{aligned} CV^2_{g1} &= \left(\tfrac{n_m}{n_p}\right)^2 CV^2_{pm} + \left(1 - \tfrac{n_m}{n_p}\right)^2 CV^2_p + \\ &2\tfrac{n_m}{n_p}\left(1 - \tfrac{n_m}{n_p}\right)(w_pCV^2_p + w_mCV_pCV_m\rho_{pm}) \end{aligned} \quad (16)$$

where w_p, w_m and CV_{pm} are given by Eqs. (9)–(11). With this, Eq. (14) may be rewritten in the simple form

$$CV^2_g = CV^2_{g0} + (CV^2_{g1} - CV^2_{g0})\rho_s \quad (17)$$

which immediately shows that $CV_{g0} \le CV_g \le CV_{g1}$. Setting n_m/n_p = 0 in Eqs. (15) and (16) results in $CV^2_{g0} = CV^2_p/n_p$ and $CV^2_{g1} = CV^2_p$, which correspond to the pile group resistance uncertainties if no monitoring is performed. In contrast, for full monitoring n_m/n_p = 1 giving $CV^2_{g0} = CV^2_{pm}/n_p$ and $CV^2_{g1} = CV^2_{pm}$.

Eq. (17) gives CV_g, which is the estimation uncertainty of pile group resistance and equivalent to the coefficient of variation of many ratios R_{gt}/R_g, where R_{gt} is a true pile group resistance from load testing and R_g the respective estimate from Eq. (12). Hence, if a sufficient number of load tests on entire pile groups is available to reliably infer CV_g from observed data, Eq. (17) may be inverted to determine a value of ρ_s. For n_m = 0 (i.e., all pile resistance uncertainties are the same) this yields

$$\rho_s = \frac{\frac{CV^2_g}{CV^2_p} - \frac{1}{n_p}}{1 - \frac{1}{n_p}} \quad (18)$$

where CV_p may be interpreted as an average pile prediction uncertainty (of an arbitrary method) applicable to the piles in the load tested groups.

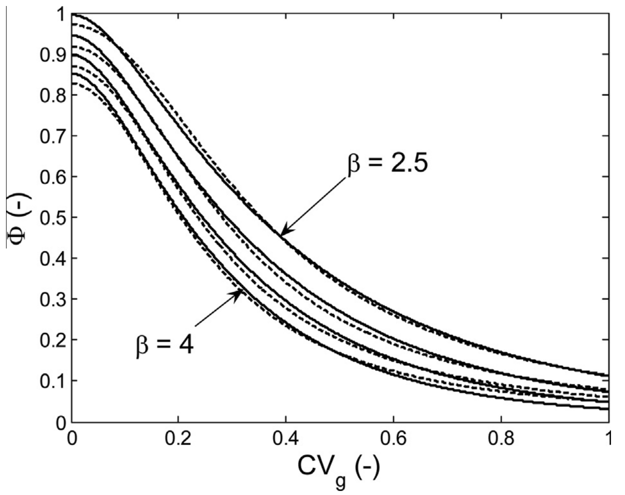

Fig. 1. LRFD Φ as a function of pile group resistance uncertainty CV_g for different pile group reliabilities β = {2.5, 3, 3.5, 4} and resistance bias factor λ_R = 1. Exact solution from Eqs. (14) and (15) (continuous) and approximation from Eq. (16) (dashed).

2.4. LRFD Φ and pile driving criteria

Knowing the expected pile group resistance R_g and uncertainty CV_g from above, and assuming a log-normal distribution of true pile group resistance about R_g, the LRFD resistance factor Φ for target reliability β and given load parameters may be found from [1,6]

$$\Phi = \frac{\lambda_R\left(\gamma_D \frac{Q_D}{Q_L} + \gamma_L\right)\sqrt{\frac{1+CV_Q^2}{1+CV_g^2}}}{\left(\lambda_{QD}\frac{Q_D}{Q_L} + \lambda_{QL}\right)\exp\left\{\beta\sqrt{\ln[(1+CV_g^2)(1+CV_Q^2)]}\right\}} \tag{19}$$

$$CV_Q^2 = \frac{\left(\frac{Q_D}{Q_L}\lambda_{QD}CV_{QD}\right)^2 + \left(\lambda_{QL}CV_{QL}\right)^2}{\left(\frac{Q_D}{Q_L}\lambda_{QD} + \lambda_{QL}\right)^2} \tag{20}$$

The term CV_Q hereby denotes the coefficient of variation of the random load, which is also assumed to be log-normally distributed. The remaining parameters in Eqs. (19) and (20) may be chosen according to AASHTO [6]. In the present work the recommended values for load cases I, II, and IV are adopted: dead load factor γ_D = 1.25, live load factor γ_L = 1.75, dead-to-live load ratio Q_D/Q_L = 2, dead load bias factor λ_{QD} = 1.08, live load bias factor λ_{QL} = 1.15, dead load coefficient of variation CV_{QD} = 0.128 and live load coefficient of variation CV_{QL} = 0.18. The resistance bias factor λ_R in Eq. (19) is set to one, since bias correction is already incorporated in Eq. (7). Note that, given log-normal pile resistances (or estimation errors thereof), the assumption of log-normal pile group resistance (or estimation error thereof) is an approximation. However, although it is known that sums of log-normal variables are not strictly log-normal, it is a reasonable approximation for sums of identically (and possibly correlated) log-normal variables. This is known as "permanence of log-normality" ([18, p. 433] and [20]) and does not contradict the Central Limit Theorem, since the log-normal distribution approaches the normal distribution for decreasing coefficients of variation (e.g., due to summing or averaging).

In order to simplify design equations for practical use, an approximation to Eq. (19) is presented in the form of

$$\Phi \approx \frac{1}{0.73 + 0.12\beta - (11 - 7.5\beta)CV_g^2} \overset{\text{for } \beta=3}{=} \frac{1}{1.09 + 11.5CV_g^2} \tag{21}$$

Eqs. (19) and (21) are shown in Fig. 1 as continuous and dashed lines, respectively, for different values of β. It is seen that the approximation is mostly conservative with a maximum absolute error of approximately 0.02. Following the LRFD principles and given a nominal design load Q_{des} defined as the expectation of the load distribution, the uniform nominal resistance of all piles in a group is obtained as $R_n = Q_{des}/(\Phi n_p)$. With this, the criteria for stopping pile advance may be defined as $R^*_{pm} = R_n$ (Eq. (7)) for monitored piles and $R_{p*} = \lambda_p R_{p'} = R_n$ (Eq. (1)) for unmonitored piles. In other words, pile driving is stopped when the estimated pile resistance reaches the nominal value R_n.

2.5. Pile groups with previously installed piles

Since some or all piles of the preliminary test program may become part of the constructed pile groups, the estimated resistances of the previously driven piles must be accounted for in the constructed resistance of the remaining piles in the same group. Since test piles are typically driven to larger resistances than the anticipated requirement, this means that the resistances of the remaining piles may be reduced. Denote by n_0 the number of previously installed piles in a group, such that $n_m - n_0$ is the number of monitored piles still to be driven (i.e., n_m remains the number of monitored piles from both preliminary and future driving) and $n_p - n_m$ is the number of unmonitored piles still to be driven. Since all previously driven piles are monitored, their combined resistance may be estimated by $\sum_{i=1}^{n_0} R^*_{pmi}$. In what follows, all previously driven piles in a group are assumed to possess a uniform nominal resistance R_{n0}. Together with the monitored and unmonitored piles not yet installed, and which are assumed to be driven to a uniform nominal resistance R_n, this results in an estimated group resistance of

$$R_g = \sum_{i=1}^{n_0} R^*_{pmi} + \sum_{i=n_0+1}^{n_m-n_0} R^*_{pmi} + \sum_{i=n_m+1}^{n_p} R^*_{pi} = n_0 R_{n0} + (n_p - n_0)R_n \tag{22}$$

This is a generalization of Eq. (12) and CV_g from Eq. (14) may be generalized to $CV_{g,pre}$ as

$$CV_{g,pre}^2 = \frac{n_p^2 R_n^2 CV_g^2 + n_0^2 CV_{pm}^2 (R_{n0}^2 - R_n^2)}{[n_0 R_{n0} + (n_p - n_0)R_n]^2} \tag{23}$$

where the numerator is the estimation variance for n_0 = 0 corrected for the fact that n_0 previously driven piles have a variance $n_0^2 R_{n0}^2 CV_{pm}^2$ rather than $n_0^2 R_n^2 CV_{pm}^2$. Eq. (23) uses a covariance between piles of different nominal resistances R_n and R_{n0} equal to the covariance, if both piles shared the same nominal resistance R_n. For the typical case of $R_{n0} > R_n$ this means that the estimation error associated with the additional resistance $R_{n0} - R_n$ is not correlated to the estimation error for pile resistance up to R_n.

Eq. (14) possesses the convenient property of being independent of R_n, such that a design value of LRFD Φ may be directly computed through Eqs. (19) and (21) or Fig. 1. This is no longer the case with Eq. (23) and, for known values of n_0 and R_{n0}, it is suggested to determine Φ through iteration by picking a starting value for Φ (e.g., 0.5) and repeating the following steps:

(1) Use the basic LRFD relationship to find $R_g = Q_{des}/\Phi$.
(2) Compute R_n from Eq. (22).
(3) Compute $CV_{g,pre}$ from Eq. (23).
(4) Find Φ from Eqs. (19) and (21) or Fig. 1.
(5) Reinitiate iteration at (1) with resulting value of Φ and continue until Φ reaches a stable value.

In the same way as above, knowing R_n the criteria for stopping pile advance may be defined as $R^*_{pm} = R_n$ (Eq. (7)) for monitored piles and $R_{p*} = \lambda_p R_{p'} = R_n$ (Eq. (1)) for unmonitored piles.

3. Practical example

In order to demonstrate the results of Section 2, data from the installation of two driven piles in the state of Florida are used with pile resistances determined from the Davisson criterion. Monitoring was performed by the PDA/CAPWAP method, while pile resistance predictions were obtained from the modified Gates equation given in Eq. (1). Depth profiles of R_m and a scatter plot of R_m versus R_p are shown in Fig. 2. In the scatter plot and all subsequent analysis data with R_m < 0.5 MN were discarded, since R_n > 1 MN is anticipated in the present example and to avoid division by zero in the computation of E_{pm} for Eq. (6). The linear regression fit of the remaining 70 data points in Fig. 2b confirms the visual impression that the data cloud is roughly distributed about a straight line, which, however, does not pass through the origin. As a consequence, the proportional bias model of Eqs. (2) and (3) is not entirely appropriate and instead of directly using R_p from Eq. (1) as the predicted resistance, we use a site-specifically adjusted version $R_p' = 0.84R_p - 0.38$ based on the linear regression fit. This leads to $E[R_m] = E[R_p] = 1.40$ by taking the respective sample means and, knowing λ_m = 1.16 from Paikowsky [1], it yields λ_p = 1.16 by Eq. (4). Data further deliver CV_{Rm} = 0.32 and CV_{Rp} = 0.29 as the sample coefficients of variation and, in combination with CV_m = 0.34 from Paikowsky [1], Eq. (5) gives CV_p = 0.37. Computing E_{mp} = 1.00 as the mean of all ratios $\lambda_m R_m/(\lambda_p R_p)$ and E_{pm} = 1.03 as the mean of all ratios $\lambda_p R_p/(\lambda_m R_m)$, Eq. (6) yields ρ_{pm} = 0.88. By Eqs. (9) and (10) this translates into estimation weights w_p = 0.20 and w_m = 0.80, further resulting in CV_{pm} = 0.34 from Eq. (11).

At the example site, single pile load test data for evaluation of ρ_s through Eq. (13) is not available. We opt to apply Eq. (18) with an empirical value of CV_g inferred from a series of load test results on pile groups at different sites of mainly n_p = 9 piles compiled by Zhang et al. [16, Table 2]. They report a maximum (most conservative) CV_g = 0.24 for freestanding pile groups in cohesionless soils. Without explicit information about the resistance estimation methods involved in the load tests an exemplary value of CV_p = 0.35 is adopted [16, Table 1] as the resistance prediction uncertainty for single piles. With this, Eq. (18) delivers ρ_s = 0.40. Furthermore, partially based on results of Zhang et al. [16] and a survey of common practice/state-of-the-art, Paikowsky [1] recommends a target reliability of β = 3 for pile groups and a reduced β = 2.33 for single piles in redundant ($n_p \geq 5$) pile groups. Thus, applying Eq. (21) to a single pile with an exemplary value of $CV_g = CV_p$ = 0.35 and β = 2.33 results in a design value of Φ = 0.55. This leads to a pile group uncertainty CV_g = 0.25, obtained from inversion of Eq. (21) with β = 3 and Φ = 0.55 (i.e., equal design based on pile and group reliabilities). In other words, the recommended reliability reduction from β = 3 to β = 2.33 between pile groups and redundant piles is equivalent to an assumed uncertainty reduction from a chosen CV_p = 0.35 for single piles to CV_g = 0.25 for pile groups. By Eq. (18) this leads to an estimate of ρ_s = 0.44 for n_p = 9, which is in reasonable agreement with ρ_s = 0.40 from above. In fact, Eq. (18) demonstrates that ρ_s increases with n_p to reach an asymptotic value of $\rho_s = CV_g^2/CV_p^2 \approx 0.50$, which is adopted in the sequel as a conservative upper bound for both predicted and monitored resistance estimation errors. The intermediate value of ρ_s between zero and one also indicates that portions of spatial variability of estimation errors are contained within pile groups as well as between pile groups.

To continue the example, suppose that a pile group consists of 5 piles with monitoring on two piles (i.e., n_p = 5 and n_p = 2) and that the design load Q_{des} = 5 MN for the group with a target reliability of β = 3. Eqs. (15)–(17) and (21) immediately yield CV_{g0} = 0.16, CV_{g1} = 0.35, CV_g = 0.27 and Φ = 0.52. Using the fundamental LRFD design equation we arrive at $R_n = Q_{des}/(\Phi n_p)$ = 1.93 MN for the nominal resistance of a single pile. The criteria for stopping pile advance are, thus, found as $R^*_{pm} = w_p\lambda_p R_p\prime + w_m\lambda_m R_m = 0.19R_p + 0.93R_m - 0.09 = R_n$ for monitored piles and $R_{p*} = \lambda_p R_p\prime = 0.97R_p - 0.44 = R_n$ for unmonitored piles. Furthermore, we consider the scenario where one pile of the group was already driven to a resistance $R_{n0} = R^*_{pm} = 2.6$ MN during the test pile program. Of the remaining four piles, one is to be monitored, such that n_p = 5, n_m = 2 and n_0 = 1 apply. Using the iterative procedure of Section 2.5 we arrive at Φ = 0.55 and R_n = 1.63 MN. With this, the driving criteria of the remaining piles may be defined analogous to above. It may be observed that in spite of a rather small change in Φ the latter values of R_n are significantly smaller than those obtained for n_0 = 0. This may be attributed to the fact that the presence of a rigid pile cap is assumed to perfectly redistribute excess loads from failed piles to still intact piles (e.g., stronger test piles). It is noted that if R_{n0} happens to be equal to R_n for n_0 = 0, then the iteration process of Section 2.5 delivers identical results to those of the direct solution in Section 2.3.

4. Discussion of results

Fig. 3 graphically represents further results in terms of LRFD Φ as a function of the degree of monitoring n_m/n_p, which are based on the practical example in combination with the parameters given in each chart. The circles correspond to the hypothetical scenario of a single pile in a group (n_p = 1) and represent minimum values of Φ. Asterisks are used for n_p = 5 and the continuous lines correspond to maximum values of Φ that are approached as n_p becomes very large (n_p >> 1). It may be consistently observed that Φ grows approximately linear with n_m/n_p between a minimum and a maximum value. These bounds correspond to a maximum group uncertainty $CV^2_{g,\max} = CV^2_p[1/n_p + (1 - 1/n_p)\rho_s]$ and a minimum group uncertainty $CV^2_{g,\min} = CV^2_{pm}[1/n_p + (1 - 1/n_p)\rho_s] = CV^2_{g,\max}CV^2_{pm}/CV^2_p$, which are obtained from Eqs. (15)–(17) by using n_m/n_p = 0 and 1, respectively. Fig. 3a shows that Φ is relatively constant with n_m/n_p. This may be attributed to the large value of ρ_{pm} = 0.88 resulting in an insubstantial uncertainty reduction through monitoring. Mathematically, this is reflected by $CV_{pm} \approx CV_p$ and $CV_{g,max} \approx CV_{g,min}$. In other words, the high correlation between the estimation errors of predicted and monitored resistances observed in the practical example renders pile monitoring largely redundant with respect to prediction equations. As a consequence, the benefit of (additional) monitoring is limited and may not be worth the cost. However, this observation should not be arbitrarily generalized, since ρ_{pm} in the practical example is inferred from depth profiles of only two piles, such that the ratios R_m/R_p and R_p/R_m may not contain the full site variability (i.e., E_{mp} and E_{pm} in Eq. (6) would be larger with more data, thus decreasing ρ_{pm}). Smaller values of ρ_{pm} also appear plausible in view of the fact that the random errors of prediction and monitoring methods may be attributed to quite different factors (e.g., hammer performance for predictions and strain sensor precision for monitoring). Moreover, ρ_{pm} near unity would imply basically identical performance of prediction equations and monitoring approaches.

Fig. 3b illustrates the effect of a decrease in ρ_{pm} to a hypothetical value of 0.1. While this does not affect Φ for n_m/n_p = 0 (no monitoring and, hence, ρ_{pm} irrelevant), the reduced correlation leads to increasingly higher values of Φ as the degree of monitoring grows. In Fig. 3c prediction uncertainty CV_p is hypothetically raised from 0.37 to 0.6. As to be expected, this leads to a lowering of Φ, which is more pronounced for low degrees of monitoring and which becomes weaker as the influence of monitoring grows. Fig. 3d demonstrates the effect of decreasing spatial correlation ρ_s from 0.5 to a hypothetical value of 0.1. For n_p = 1, ρ_s is irrelevant and no changes are observed. For n_p > 1, Φ is seen to increase more strongly, which is due to the elevated independence of estimation errors between piles and a larger degree of uncertainty reduction after summing those errors into a pile group uncertainty. For the

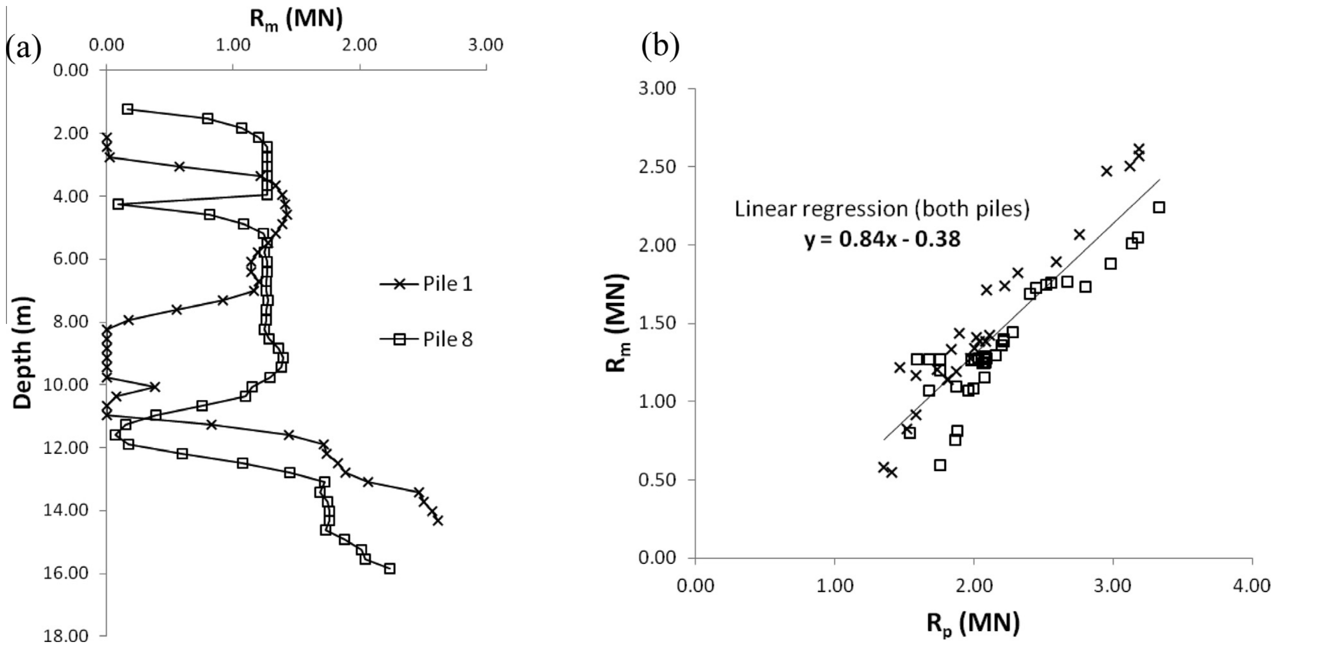

Fig. 2. Pile driving data from piles 1 and 8 of the Dixie Highway project in Florida. (a) Depth profiles of PDA/CAPWAP monitored resistances R_m and (b) scatter plot of all R_m > 0.5 MN versus R_p from modified Gates equation (Eq. (1)) with linear regression fit.

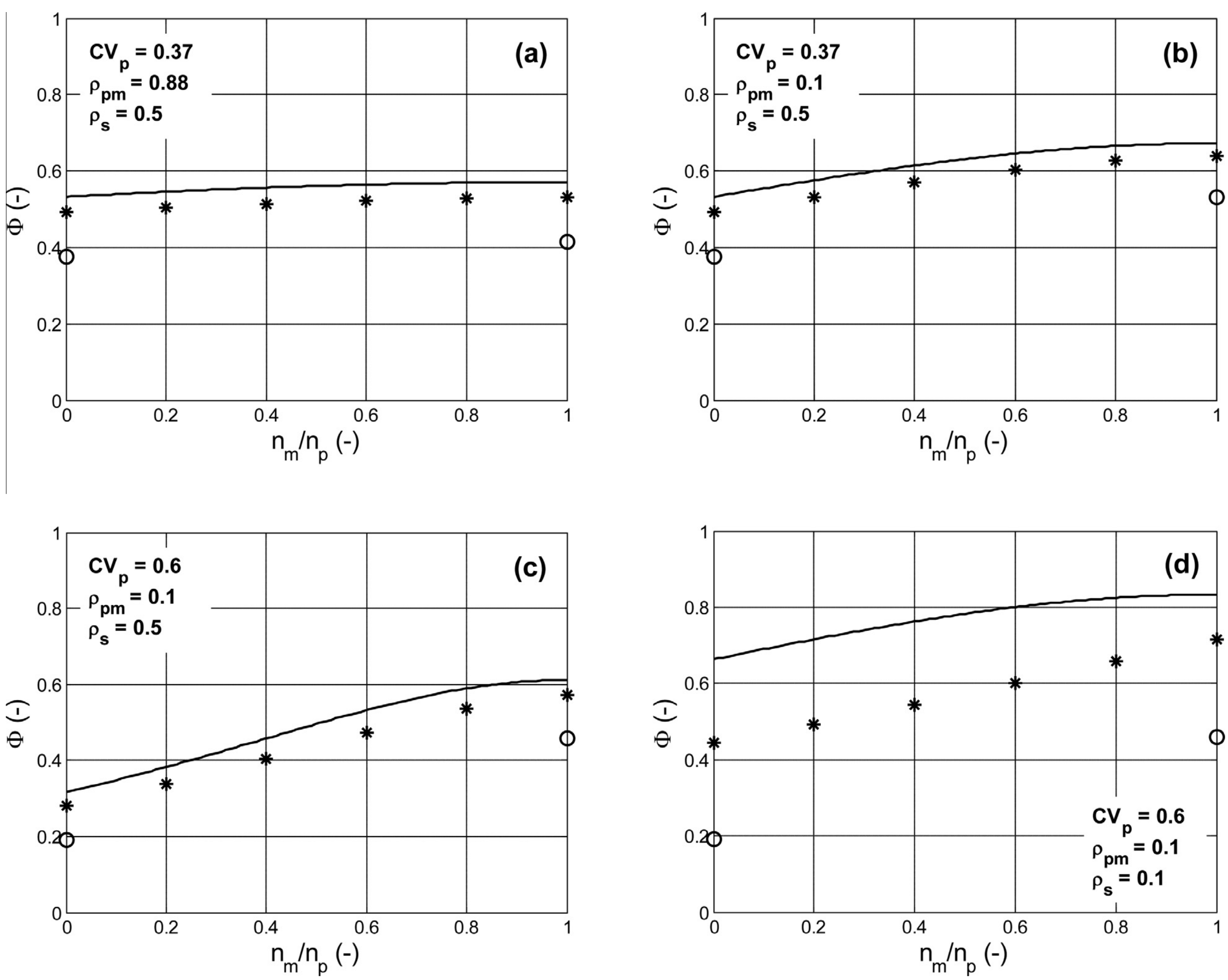

Fig. 3. LRFD Φ from Eq. (21) for β = 3 as a function of degree of monitoring n_m/n_p for the practical example in combination with the parameters indicated in each chart. Continuous lines are asymptotic maxima for n_p >> 1, asterisks are for n_p = 5 and circles are minima for hypothetical case of n_p = 1.

preliminary empirical estimate of ρ_s = 0.5 from pile group load tests and current design practice, Fig. 3a–c illustrates the convenient fact that Φ is quite close to its asymptotic maximum value for $n_p \geq 5$. This closeness increases for ρ_s > 0.5 and means that in an initial design phase a conservative minimum resistance factor Φ_{min} (corresponding to $CV_{g,max}$ for n_p = 5 and ρ_s = 0.5) and a conservative maximum resistance factor Φ_{max} (corresponding to $CV_{g,min}$ for n_p = 5 and ρ_s = 0.5) may be determined to linearly approximate $\Phi \approx \Phi_{min} + (\Phi_{max} - \Phi_{min})n_m/n_p$. This simple relationship is independent of the total number n_p of piles in a group and relates the degree of monitoring n_m/n_p directly to a value of LRFD Φ. Fig. 3a–c further illustrates that, for an inferred value of ρ_s = 0.5, maximum values of Φ for full monitoring range between approximately 0.55 and 0.65. This agrees well with current design practice as reflected by an efficiency factor Φ/λ_R = 0.56 reported by Paikowsky [1, Table 20] for β = 2.33 (single pile). It is recalled once more that the present approach applies bias correction at the very beginning, such that resulting values of Φ are identical to respective efficiency factors (λ_R = 1). The partial superiority of the present approach (larger Φ/λ_R) may be attributed to the combination of predicted and monitored resistances and, as such, becomes more pronounced for smaller values of CV_p and ρ_{pm} (Fig. 3b and c).

The proposed site-specific calibration of pile resistance predictions from dynamic equations to those of dynamic measurements (monitoring) from a test pile program attempts to ensure appropriate bias and uncertainty parameters for a site. However, hammer performance, for example, may be highly variable during pile driving within a single site [4]. That is, a successful initial calibration from a test pile program may deteriorate in the course of a pile driving job. It is recommended, therefore, that the relationship between predicted and monitored resistances be accompanied and/or up-dated as data from monitored production piles become available. In addition, validation load tests as proposed by Zhang [5] and Liang and Yang [12] may serve to gain additional confidence in reliable pile group construction over the whole site.

5. Summary

Many national construction standards require reliability based design (e.g., [6]) to assure acceptable levels of reliabilities of engineered systems. Driven pile groups are a common type of deep foundation and probabilistic load parameters as well as target reliabilities are generally defined at the pile group level, i.e., not for single piles or components thereof. Pile driving criteria, however, are required for individual piles, which is currently achieved by using different target reliabilities at the pile and group levels [1]. In the present work we consider piles under axial loading and present an approach for defining pile driving criteria for individual piles, while satisfying target reliability at the group level. Total pile resistances (side plus tip components) are estimated from dynamic predictions (e.g., Gates equation) at all piles and from dynamic monitoring (e.g., PDA/CAPWAP) at an arbitrary number of monitored piles in a group. Resistance estimates from equations may strongly depend on variable properties of the driving equipment and are site-specifically calibrated to monitored resistances from test pile programs, for example. Estimates from dynamic equations and monitoring are then combined through best linear unbiased estimation (BLUE) to arrive at improved resistance estimates of monitored piles. The latter are again combined with resistance estimates from dynamic equations at unmonitored piles to yield an estimated pile group resistance with uncertainty.

Based on this, expressions for LRFD Φ and pile driving criteria are formulated as a function of prediction uncertainty CV_p, monitoring uncertainty CV_m, correlation coefficient between estimation errors of prediction and monitoring methods ρ_{pm} and correlation coefficient ρ_s between estimation errors at different piles within a group. While CV_m and ρ_s are inferred from load test data bases, CV_p and ρ_{pm} are indirectly calibrated to monitored resistances at an exemplary site (using Davisson criterion through the FHWA modified Gates equation and PDA/CAPWAP monitoring). Results in Fig. 3 show that for $\rho_s \geq 0.5$ and five or more piles in a group ($n_p \geq 5$) Φ becomes relatively insensitive to n_p and grows approximately linear with the degree of monitoring n_m/n_p between a minimum and a maximum value. Overall, and as to be expected, Φ decreases with increasing CV_p and CV_m (larger uncertainties) as well as with ρ_{pm} and ρ_s (more data redundancy). In order to benefit from a significant increase in Φ due to monitoring both CV_m and ρ_{pm} have to be small. Besides these general results for Φ, an iterative method for reliability based pile group design in the presence of previously driven piles in the group is also presented. It allows for the flexibility to decrease nominal pile resistances of the remaining piles, if previously driven piles in the group (e.g., from test pile program) are advanced to larger depths (resistances).

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