Reliability based design of driven pile groups using combination of pile driving equations and high strain dynamic pile monitoring

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1. Introduction

Within the category of deep foundations, driven piles are a common alternative to drilled shafts. Depending on the stage of implementation, numerous methods exist for the design and quality control of driven piles. These methods may be grouped into three major categories [1]: (1) “Static equations” (e.g., Tomlinson or Schmertmann methods) using measurements of certain soil parameters, SPT and/or CPT data. (2) “Dynamic equations” (e.g., ENR or Gates equations) using information from the pile driving process, such as blow count and hammer properties. (3) “Dynamic measurements” (e.g., Case method or Case Pile Wave Analysis Program – CAPWAP) using data from pile driving analyzer (PDA), i.e., high strain dynamic testing (HSDT). More recently, embedded data collectors (EDC) are available as an alternative to PDA with gages at the bottom as well as at the top of the pile, and results compare well to PDA/CAPWAP [2] as well as static load tests [3]. In a typical driven pile project (1) is applied in the pre-construction phase for pile design, while (2) and (3) are used as pile driving criteria during construction for stopping pile advance and may be adjusted according to static validation load tests on a number of piles [4,5].

Reliability based design (e.g., load and resistance factor design – LRFD; [1,6]) aims at achieving a prescribed level of reliability (maximum permissible probability of failure) for engineered systems. This requires a conceptual model, which is consistent with the design and construction procedure (e.g., the pile driving criteria) and accounts for all types of uncertainties involved (e.g., uncertainties of prediction methods, soil parameters or due to spatial variability of ground properties within a site). Many approaches make use of pile load test databases to determine reliabilities of existing foundations or to establish LRFD resistance factors for future design purposes. Under axial loads, these approaches include Allen [7] for dynamic pile driving equations as well as Kwak et al. [8] for design based on SPT data. McVay et al. [9], Paikowsky [1] and Yoon et al. [10] investigate reliability based performance of a series of different static and dynamic methods, while Zhang [5], Zhang et al. [11] and Liang and Yang [12] propose the use of dynamic measurements and static load tests for reliability based quality control at the end of the construction phase. In an effort to reduce design uncertainty based on large and possibly heterogeneous load test databases, Zhang et al. [13] apply Bayesian updating to incorporate less variable regional and site-specific information. Alternative ap-
proaches combining uncertainties of relevant design parameters by propagation through design equations rather than comparing predictions against load test results are taken by Foye et al. [14] and Kim et al. [15]. These methods do not rely on the compilation of pertinent load test databases, however, they may not fully account for uncertainties of the design equations (method error) and statistical properties of some design parameters (including correlations between parameters) may not always be well defined.

While the vast majority of reliability studies focus on individual piles (or even side and tip resistances separately), it is logical that target reliabilities should apply to entire systems, i.e., containing the superstructures [16]. However, due to the generally large complexity of the problems (e.g., an entire bridge with many elements influencing each other to a smaller or larger degree), super and substructure design are widely decoupled in the sense that geotechnical engineers receive prescribed target reliabilities and load properties for each foundation element (e.g., one driven pile group). It has been recognized that reliabilities at the pile and group level may be drastically different due to effects of a rigid pile cap and pile redundancy within a group (i.e., failure of a single pile does not necessarily lead to failure of the whole group). In order to account for the latter, it has been suggested and become part of design standards that assigned pile reliabilities should be smaller than required target reliabilities of pile groups [1,9,16]. In the present work, we establish a methodology, which aims to achieve a given target reliability at the pile group level by defining pile driving criteria for individual piles based on the number of piles in a group and a combination of pile resistance predictions from dynamic equations and an arbitrary number of dynamic measurements. We hereby consider the predictions from dynamic measurements as reliably calibrated to static load test databases (e.g., [1]) and perform site-specific calibration of dynamic equations with respect to dynamic measurements. This is to reduce the influence of variable driving equipment performance between sites [4]. While the following sections deal with single piles and the calibration and combination of predictions from dynamic equations and measurements, subsequent sections focus on the pile group level and the development of LRFD resistance factors. A practical example problem and charts are presented thereafter to illustrate the findings.

2. Theoretical development

2.1. Site specific calibration of prediction equation

For the purpose of illustrating the statistical concept, assume a large site with many statically load tested driven piles, such that all “true” pile resistances \( R_p \) are random, but known. Note that herein, pile resistance always refers to the total axial resistance in general, which is the sum of side plus tip contributions. For every pile it is generally possible to make a prediction of \( R_p \) by one of the available dynamic equations. This will be called the “predicted value” \( \bar{R}_p \) in the sequel and it is a random variable with bias and uncertainty with respect to \( R_p \). For example, if the Federal Highway Administration (FHWA) modified Gates equation [1] is used, one obtains after conversion to SI units

\[
R_p = 0.021 \sqrt{WH} \log_{10}(0.254N) - 0.445
\]  

(1)

\( A, B, C, \mu \) auxiliary variables in dimension of force

\( E[\cdot] \) expectation operator

\( H \) hammer stroke in meters

\( N \) blow count in blows per meter

\( W \) hammer weight in kilograms
where \( R_p \) is the Davison resistance in MN, \( W \) is the hammer weight in kilograms, \( H \) the hammer stroke in meters and \( N \) the blow count in blows per meter. We further define a “monitored value” \( R_m \) of \( R_p \), which is also random and obtained from possible dynamic measurements (e.g., PDA or EDC) during pile driving. Through a comprehensive database analysis of load tests and using the Davison criterion, \( R/R_m \) from using PDA with CAPWAP analysis, for example, has been found to have an expectation (LRFD “biais”) of \( \lambda_{m} = 1.16 \) with a coefficient of variation \( CV_m = 0.34 \) \cite{1}.

Analogous values of bias \( \lambda_p \) and uncertainty \( CV_p \) for \( R_p \) are also available; however, variability in hammer performance between and even during particular driving jobs may be significant \cite{4}. In order to control this variability at least between different sites without significant increase in cost due to static load testing, \( \lambda_m \) and \( CV_m \) are assumed to be reliably known and we propose to calibrate \( R_p \) indirectly to \( R_m \) rather than directly to \( R_p \). Respective data are typically available as depth profiles of \( R_p \) and \( R_m \) from preliminary test pile programs at a site. According to LRFD practice the relationships

\[
R_l = \lambda_p R_p e_p
\]

\[
R_l = \lambda_m R_m e_m
\]

apply, where \( \lambda_p \) and \( \lambda_m \) are random error terms (independent of \( R_p \) and \( R_m \), respectively) of unit expectation and coefficients of variation \( CV_p \) and \( CV_m \). Since both estimators are unbiased, the expectations of the right-hand-sides of Eqs. (2) and (3) have to be equal to the expectation of \( R_p \), leading to

\[
\lambda_p = \frac{\lambda_m E[R_m]}{E[R_p]}
\]

may be obtained with \( E[\cdot] \) denoting the expectation operator. Knowing \( \lambda_m \) from historical data and \( E[R_m] \) and \( E[R_p] \) from the sample means of observed data, \( \lambda_p \) may be inferred. Furthermore, the coefficients of variation of Eqs. (2) and (3) may be written as \( CV^2_p + CV^2_m = CV^2_\lambda \), where \( CV_p \) and \( CV_m \), respectively, where \( CV_p \) and \( CV_m \) are the coefficients of variation of observed values of \( R_p \) and \( R_m \). Both Eqs. (2) and (3) have to reproduce the coefficient of variation of \( R_p \) leading to

\[
CV^2_p = CV^2_m + CV^2_\lambda - CV^2_{\lambda_p}
\]

with \( CV_m \) again known from historical data. In analogy to standard regression theory, \( CV_{\lambda_p} \) (or \( CV_{\lambda_m} \)) may be regarded as the portion of variability in \( R_l \) explained by the estimator, while \( CV^2_p \) (or \( CV^2_m \)) represents the complementary portion not explained by the estimator.

The bias corrected estimators \( \lambda_p R_p \) and \( \lambda_m R_m \) are expected to be correlated to each other as their common purpose is to closely reproduce variable values of \( R_l \). The respective relative estimation errors \( \epsilon_p \) and \( \epsilon_m \) of any fixed \( R_l \) may also possess a non-zero correlation coefficient \( \rho_{\lambda_m \lambda_p} \) to be determined. Eliminating \( R_l \) from Eqs. (2) and (3) gives \( \lambda_p R_p \cdot \lambda_m R_m \) which possesses an expectation \( E_{\lambda_m \lambda_p} \) equal to \( E[R_l/R_m] E[R_p/R_m] = E[\epsilon_p]/E[\epsilon_m] = 1 + CV^2_\lambda - CV^2_{\lambda_p} \). The left-hand-side is found using Eq. (4) and may be evaluated from field observations of \( R_p \) and \( R_m \). The right-hand-side is derived by Journel and Huijbregts \cite{17, p. 426} using a low order approximation. For log-normal \( \epsilon_p \) and \( \epsilon_m \) in combination with \( \rho_{\lambda_m \lambda_p} = 0 \) it can be shown \cite{18, p. 103 and 6451} that this approximation becomes exact. In the same way, an expectation \( E_{\lambda_m \lambda_p} \) for the reciprocal may be expressed as \( E[R_p/R_m] = E[R_p/R_m] = E[\epsilon_p]/E[\epsilon_m] \). With this, the average \( (E_{\lambda_m \lambda_p} + E_{\lambda_m \lambda_p})/2 \), for example, may be obtained from observations and used to infer \( \rho_{\lambda_m \lambda_p} \) as

\[
\rho_{\lambda_m \lambda_p} = \frac{1 - 0.5(CV^2_\lambda + CV^2_m - E_{\lambda_m \lambda_p} - E_{\lambda_m \lambda_p})}{CV_m CV_p}
\]

where it is recalled that \( E[\epsilon_p] = E[\epsilon_m] = 1 \). In summary, in the above development it is assumed that \( \lambda_m \) and \( CV_m \) are reliably known from databases. This immediately implies that the expectation and coefficient of variation of \( R_p \) are also known reliably from Eq. (3), if sufficient observations of \( R_m \) are available. Based on this knowledge Eqs. (4) and (5) allow inferring \( \lambda_p \) and \( CV_p \), from a series of observed values of \( R_m \) and \( R_p \) at a site (e.g., from depth profiles of preliminary test piles). Similarly, Eq. (6) explores the observed expectations \( E_{\lambda_m \lambda_p} \) to infer \( \rho_{\lambda_m \lambda_p} \); the use of \( (E_{\lambda_m \lambda_p} + E_{\lambda_m \lambda_p}) \) as opposed to \( E_{\lambda_m \lambda_p} \) or \( E_{\lambda_m \lambda_p} \) alone is hereby thought to avoid ambiguity (possibly different values of \( \rho_{\lambda_m \lambda_p} \) based on \( E_{\lambda_m \lambda_p} \) or \( E_{\lambda_m \lambda_p} \) alone). Most importantly, neither of Eqs. (4)–(6) requires values of true load test resistances \( R_p \).

It is noted, however, that the bias model of Eqs. (2) and (3) is purely proportional, i.e., it is based on the assumption that scatter-plots of \( R_p \) versus \( R_p \) and \( R_m \) versus \( R_m \) form clouds around a straight line through the origin. As a consequence, data points of \( R_m \) versus \( R_p \) should also scatter around a straight line through the origin. If preliminary data indicates that this is not the case at a given site, then the prediction equation for \( R_p \) (e.g., Eq. (1)) should be revised to achieve this proportionality. A modified estimator \( R_p = a + bR_m \) based on standard linear regression between \( R_p \) and \( R_m \) can be used in Eqs. (4)–(6) instead of \( R_p \) (see practical example in Section 3). The adjustment by linear regression then implies that \( E[R_p]/E[R_m] = 1 \) and \( CV_{\lambda_p} < CV_{\lambda_m} \), which simplifies Eqs. (4) and (6) and assures \( CV_p > CV_m \) in Eq. (5).

### 2.2. Combining predicted and monitored resistances of a single pile

Using the two unbiased estimators \( R_p = \lambda_p R_p \), \( R_m = \lambda_m R_m \) and of \( R_p \) with their respective uncertainties, a best linear unbiased estimator (BLUE; \cite{19, p. 278}) of the form

\[
R_{pm} = w_p R_p + w_m R_m
\]

is used with weights \( w_p \) and \( w_m \) to obtain an optimal (in the sense of unbiasedness and minimum error variance) estimate \( R_{pm} \) of \( R_p \). In spatial interpolation this is equivalent to solving the Ordinary Kriging system, which is here formulated based on estimation error variances,

\[
\begin{bmatrix}
E[R^2_p] & E[R_p R_m] & 1 & 1 & 0 & \mu \n
E[R^2_m] & E[R_p R_m] & E[R^2_m] & 1 & 0 & \mu
1 & 0 & 1 & 1 & 1
\end{bmatrix} \begin{bmatrix}
w_p \n
w_m \n
\mu
\end{bmatrix} = \begin{bmatrix}
0 \n
0
\end{bmatrix}
\]

and where \( E[R^2] \) is used as the common expectation of \( R_p \) and \( R_m \). Thus, the products on the diagonal of the coefficient matrix represent the variances of estimation errors \( \epsilon_p \) and \( \epsilon_m \) and the off-diagonal product represents the respective covariance. Parameter \( \mu \) is a Lagrangian operator, which is not further required in the sequel, and the zeros on the right-hand-side are due to independence between estimation errors and true resistances. Solving Eq. (8) gives

\[
w_p = \frac{CV^2_m - CV^2_m CV_p \rho_{\lambda_m \lambda_p}}{CV^2_p + CV^2_m - 2CV_m CV_p \rho_{\lambda_m \lambda_p}}
\]

\[
w_m = \frac{CV^2_p - CV^2_m CV_p \rho_{\lambda_m \lambda_p}}{CV^2_p + CV^2_m - 2CV_m CV_p \rho_{\lambda_m \lambda_p}}
\]

with an estimation uncertainty \( CV^2_{pm} = w_p^2 CV^2_p + w_m^2 CV^2_m + 2w_p w_m CV_p CV_m \rho_{\lambda_m \lambda_p} \) of \( R_{pm} \) as

\[
CV^2_{pm} = \frac{CV^2_p CV^2_m (1 - \rho_{\lambda_m \lambda_p}^2)}{CV^2_p + CV^2_m - 2CV_m CV_p \rho_{\lambda_m \lambda_p}}
\]

It can be shown that \( CV^2_{pm} \) is always less than or equal to both \( CV^2_p \) and \( CV^2_m \), which illustrates the benefit of using a combination of two estimates for \( R_p \). This benefit increases as the two estimates become more independent (\( \rho_{\lambda_m \lambda_p} \) approaching zero). It is further noted that Eqs. (9)–(11) for \( \rho_{\lambda_m \lambda_p} = 0 \), and when expressed in variances rather than \( CV_p \), are formally identical to results of a Bayesian updating approach by
Zhang et al. [13] between global and regional data. Finally, for unmonitored piles, i.e., when \( R_p \) is not available, \( CV_{nm} \) may be set to a very large value leading to the trivial results of \( w_p = 1, w_m = 0 \) and \( CV_{pm} = CV_p \). In cases when a number of piles is monitored, but \( \mu_{pm} \) is not reliably known or close to one, it is a conservative approximation to disregard predicted resistances at monitored piles, i.e., to use \( CV_p \) for \( CV_m \) >> 1 leading to \( w_p = 0, w_m = 1 \) and \( CV_{pm} = CV_m \).

2.3. Pile groups with no previously installed piles

Consider a pile group of \( n_p \) piles, all of which are still to be driven and where an arbitrary number \( n_m \) of which are to be monitored. Using \( R_g \) and \( R_{pm} \) to estimate resistances of unmonitored and monitored piles in the group, respectively, an unbiased estimate of total group resistance \( R_e \) is obtained as the sum of each pile’s resistance estimate (considering a group efficiency factor of one; [16]). Due to unbiasedness, \( R_g \) represents the true pile resistance at a particular location and it is, hence, equivalent to the associated “nominal” pile group resistance in LRFD context.

\[
R_e = \sum_{i=1}^{n_p} R_{pm}^i + \sum_{i=n_m+1}^{n_p} R_{pi} = n_p R_g
\]  

(12)

This assumes no contribution of the pile cap and that pile spacing is large enough (e.g., over two times pile diameter), such that the possibility of block failure may be excluded [16]. It is further reasonable to assume that all piles are driven to the same nominal resistance \( R_g \), such that the final expression of Eq. (12) is valid. Hereby, and in analogy to \( R_g \), \( R_{pm} \) is the unbiased estimate (expectation) of true pile resistance given by \( R_p \) and \( R_{pm} \) for unmonitored and monitored piles, respectively. The next goal is to find the value of \( R_{pm} \) such that driving criteria for individual piles (monitored and unmonitored) may be established, while satisfying the target reliability at the group level.

For combining pile resistance uncertainties into a group resistance uncertainty it is necessary to consider possible correlation of estimation errors between different piles in a group. Zero or small correlation may be expected if estimation errors are mainly caused by random factors that are independent from pile to pile (e.g., instrumentation/operation errors, rapidly changing ground conditions, etc.). Stronger correlations will occur if estimation errors are influenced by site or driving conditions, which vary between pile groups, but are relatively constant for a single pile group. Hence, in general, estimation error may be regarded as a spatially random function possessing a spatial covariance function (or variogram) with a particular correlation length [17–19]. However, it is unlikely to obtain sufficient load test data at a site for calculating estimation errors and their full variogram for exact evaluation of pile group uncertainty. As an alternative, an average spatial correlation coefficient \( \rho_s \) is introduced, which expresses the average degree of correlation between estimation errors of different piles in a group. Similar to Eq. (6), \( \rho_s \) may be determined from the relationship \( E[i_i/s] = E[i_i/E_s] (1 + CV_p^2 - CV_{np} CV_{pm} \rho_s) \), where \( i_i \) and \( s \) are known estimation errors (from Eq. (2) or (3)) at two different piles, which are not separated further than the maximum pile separation distance in a production pile group. In other words, \( E[i_i/s] \) may be evaluated by pairing up all possible combinations of load tested piles, eliminating those pairs that are separated by more than the maximum pile distance in a group and computing the empirical mean of all remaining ratios \( i_i/s \). Note hereby, that piles paired up with themselves (zero separation distance) are not included and that each valid pile pair contributes two ratios \( i_i/s \) and \( i_j/s \) to the mean (which assures \( E[i_i/s] > 1 \)). Knowing also that \( E[i] = E[i_j] \) and \( CV_{np} = CV_s \) it is found that

\[
\rho_s = 1 - \frac{E[i_i/s] - 1}{CV_s^2}
\]  

(13)

Note that this method of estimating \( \rho_s \) is analogous to using a single point of the experimental variogram, except for exploring \( E[i_i/i_j] \) instead of \( E[i_i/i_j] \), where \( E[i_i/i_j] \) and Eq. (13) may be applied to both predicted \( CV_p = CV_{np} \) and monitored \( CV_p = CV_{pm} \) resistance errors. However, practical applicability may still be limited by requiring a sufficient number of load tests (e.g., \( > 20 \) load test pairs equivalent to 7 load tests in one group) at short separation distances. As an alternative, based on the definition of \( \rho_s \) given here, it will be derived further below in terms of load test results on entire pile groups (rather than separate nearby single piles). For this purpose, the resistance estimation uncertainty of pile groups is required.

The variance of a sum is known to be equal to the sum of all elements in the variance–covariance matrix of the summands. Applying this to \( R_g \) from Eq. (12) we obtain a symmetric \( n_p \) by \( n_p \) matrix and a pile group uncertainty \( CV_s^2 \) after division by \( R_g^2 \) of

\[
CV_s^2 = \frac{1}{n_p} (A + B + C)
\]

(14)

where

\[
a = n_m CV_{pm}^2 + (n_p - n_m) CV_p^2
\]

\[
b = n_m(n_m - 1)CV_{pm}^2 \rho_s + (n_p - n_m)(n_p - n_m - 1) CV_p^2 \rho_s
\]

\[
c = 2n_m(n_p - n_m)(w_p CV_p^2 + w_m CV_m CV_{pm} \rho_s) \rho_s
\]

The two terms in \( A \) correspond to the sums of the variances of \( n_m \) monitored and \( n_p - n_m \) unmonitored piles. The first and second terms in \( B \) are the sums of all covariances between monitored and unmonitored piles, respectively. Finally, \( C \) represents the sum of all covariances between monitored and unmonitored piles. Eq. (14) implies that \( \rho_s \) applies to predicted and monitored estimation errors and that the correlation between a prediction error at one pile location and a monitored resistance error at another pile location is equal to \( \rho_{pm} \). Denoting by \( CV_{g0} \) and \( CV_{g1} \) the uncertainties for the limiting cases of \( \rho_s = 0 \) and \( \rho_s = 1 \), Eq. (14) leads to

\[
CV_{g0}^2 = \frac{1}{n_p} \left[ \frac{n_m}{n_p} CV_{pm}^2 + \left( 1 - \frac{n_m}{n_p} \right) CV_p^2 \right]
\]  

(15)

\[
CV_{g1}^2 = \frac{1}{n_p} \left( \frac{n_m}{n_p} \right)^2 CV_{pm}^2 + \left( 1 - \frac{n_m}{n_p} \right)^2 CV_p^2 + 2 \frac{n_m}{n_p} \left( 1 - \frac{n_m}{n_p} \right) (w_p CV_p^2 + w_m CV_m CV_{pm} \rho_{pm}) \rho_s
\]  

(16)

where \( w_p, w_m \) and \( CV_{pm} \) are given by Eqs. (9)–(11). With this, Eq. (14) may be rewritten in the simple form

\[
CV_s^2 = CV_{g0}^2 + (CV_{g1}^2 - CV_{g0}^2) \rho_s
\]  

(17)

which immediately shows that \( CV_{g0} \leq CV_s \leq CV_{g1} \). Setting \( n_m/n_p = 0 \) in Eqs. (15) and (16) results in \( CV_{g0} = CV_p^2/n_p \) and \( CV_{g1} = CV_p^2 \), which correspond to the pile group resistance uncertainties if no monitoring is performed. In contrast, for full monitoring \( n_m/n_p = 1 \) giving \( CV_{g0} = CV_{pm}^2/n_p \) and \( CV_{g1} = CV_{pm}^2 \).

Eq. (17) gives \( CV_p \), which is the estimation uncertainty of pile group resistance and equivalent to the coefficient of variation of many ratios \( R_g/R_p \), where \( R_g \) is a true pile group resistance from load testing and \( R_p \) the respective estimate from Eq. (12). Hence, if a sufficient number of load tests on entire pile groups is available to reliably infer \( CV_{g0} \) from observed data, Eq. (17) may be inverted to determine a value of \( \rho_s \). For \( n_m = 0 \) (i.e., all pile resistance uncertainties are the same) this yields

\[
\rho_s = \frac{CV_s^2 - \frac{1}{n_p}}{1 - \frac{1}{n_p}}
\]  

(18)

where \( CV_p \) may be interpreted as an average pile prediction uncertainty (of an arbitrary method) applicable to the piles in the load tested groups.
and uncertainty $\beta = 2.5, 3, 3.5, 4$ and resistance bias factor $\lambda_R = 1$. Exact solution from Eqs. (14) and (15) (continuous) and approximation from Eq. (16) (dashed).

$\Phi = \frac{\gamma_R (\gamma_D + \eta)}{\gamma_D (\gamma_D + \eta)} \exp \left( \frac{\beta}{\sqrt{1 + CV^2}} \right)$  

$CV_Q^2 = \frac{\left( \frac{\gamma_D}{\gamma_D + \eta} \right) CV_Q^2 + \left( \frac{\lambda_R CV_Q^2}{\gamma_D (\gamma_D + \eta)} \right)^2}{\left( \frac{\gamma_D}{\gamma_D + \eta} \right) CV_Q^2 + \lambda_R CV_Q^2}$  

The term $CV_Q$ hereby denotes the coefficient of variation of the random load, which is also assumed to be log-normally distributed. The remaining parameters in Eqs. (19) and (20) may be chosen according to AASHTO [6]. In the present work the recommended values for load cases I, II, and IV are adopted: dead load factor $\gamma_D = 1.25$, live load factor $\gamma_L = 1.75$, dead-to-live load ratio $Q_L/Q_Q = 2$, and live load bias factor $\lambda_Q = 1.08$, live load bias factor $\lambda_Q = 1.15$, dead load coefficient of variation $CV_{Q_D} = 0.128$ and live load coefficient of variation $CV_{Q_Q} = 0.18$. The resistance bias factor $\lambda_R$ in Eq. (19) is set to one, since bias correction is already incorporated in Eq. (7). Note that, given log-normal pile resistances (or estimation errors thereof), the assumption of log-normal pile group resistance (or estimation error thereof) is an approximation. However, although it is known that sums of log-normal variables are not strictly log-normal, it is a reasonable approximation for sums of identically (and possibly correlated) log-normal variables. This is known as “permanence of log-normality” ([18, p. 433] and [20]) and does not contradict the Central Limit Theorem, since the log-normal distribution approaches the normal distribution for decreasing coefficients of variation (e.g., due to summing or averaging).

In order to simplify design equations for practical use, an approximation to Eq. (19) is presented in the form of

$\Phi = \frac{1}{0.73 + 0.12/\beta - (11 - 7.5/\beta) CV^2_g}$  

for $\beta = 3$ 

$= \frac{1}{1.09 + 11.5CV^2_g}$  

Eqs. (19) and (21) are shown in Fig. 1 as continuous and dashed lines, respectively, for different values of $\beta$. It is seen that the approximation is mostly conservative with a maximum absolute error of approximately 0.02. Following the LRFD principles and given a nominal design load $Q_{des}$ defined as the expectation of the load distribution, the uniform nominal resistance of all piles in a group is obtained as $R_n = Q_{des}/(\Phi \eta_p)$. With this, the criteria for stopping pile advance may be defined as $R_{pm} = R_n$ (Eq. (7)) for monitored piles and $R_p = \lambda_p R_{pm} = R_n$ (Eq. (1)) for unmonitored piles. In other words, pile driving is stopped when the estimated pile resistance reaches the nominal value $R_n$.

2.5. Pile groups with previously installed piles

Since some or all piles of the preliminary test program may become part of the constructed pile groups, the estimated resistances of the previously driven piles must be accounted for in the constructed resistance of the remaining piles in the same group. Since test piles are typically driven to larger resistances than the anticipated requirement, this means that the resistances of the remaining piles may be reduced. Denote by $n_0$ the number of previously installed piles in a group, such that $n_m - n_0$ is the number of recorded piles still to be driven (i.e., $n_m$ remains the number of monitored piles from both preliminary and future driving) and $n_0 - n_m$ is the number of unmonitored piles still to be driven. Since all previously driven piles are monitored, their combined resistance may be estimated by $\sum_{i=1}^{n_0} R_{pm_i}$. In what follows, all previously driven piles in a group are assumed to possess a uniform nominal resistance $R_{np}$. Together with the monitored and unmonitored piles not yet installed, and which are assumed to be driven to a uniform nominal resistance $R_n$, this results in an estimated group resistance of

$R_g = \sum_{i=1}^{n_0} R_{pm_i} + \sum_{i=n_0+1}^{n_m} R_{pm_i} + \sum_{i=n_0+1}^{n_m} R_{pm_i} = n_0 R_{np} + (n_m - n_0) R_n$  

This is a generalization of Eq. (12) and $CV_g$ from Eq. (14) may be generalized to $CV_{g,pre}$ as

$CV^2_{g,pre} = \frac{n_0^2 CV^2_{Q_D} + n_0^2 CV^2_{Q_Q} (R_{np} - R_n)}{[n_0 R_{np} + (n_m - n_0) R_n]^2}$  

where the numerator is the estimation variance for $n_0 = 0$ corrected for the fact that $n_0$ previously driven piles have a variance $n_0^2 CV^2_{Q_D}^2$ rather than $n_0^2 CV^2_{Q_D}$. Eq. (23) uses a covariance between piles of different nominal resistances $R_p$ and $R_n$ equal to the covariance, if both piles shared the same nominal resistance $R_n$. For the typical case of $R_{np} > R_n$ this means that the estimation error associated with the additional resistance $R_{np} - R_n$ is not correlated to the estimation error for pile resistance up to $R_n$.

Eq. (14) possesses the convenient property of being independent of $R_n$, such that a design value of LRFD $\Phi$ may be directly computed through Eqs. (19) and (21) or Fig. 1. This is no longer the case with Eq. (23) and, for known values of $n_0$ and $R_{np}$, it is suggested to determine $\Phi$ through iteration by picking a starting value for $\Phi$ (e.g., 0.5) and repeating the following steps:

(1) Use the basic LRFD relationship to find $R_g = Q_{des}/\Phi$.
(2) Compute $R_n$, from Eq. (22).
(3) Compute $CV_{g,pre}$ from Eq. (23).
(4) Find $\Phi$ from Eqs. (19) and (21) or Fig. 1.
(5) Reinitiate iteration at (1) with resulting value of $\Phi$ and continue until $\Phi$ reaches a stable value.

In the same way as above, knowing $R_n$, the criteria for stopping pile advance may be defined as $R_{pm} = R_n$ (Eq. (7)) for monitored piles and $R_p = \lambda_p R_{pm} = R_n$ (Eq. (1)) for unmonitored piles.
3. Practical example

In order to demonstrate the results of Section 2, data from the installation of two driven piles in the state of Florida are used with pile resistances determined from the Davison criterion. Monitoring was performed by the PDA/CAPWAP method, while pile resistance predictions were obtained from the modified Gates equation given in Eq. (1). Depth profiles of $R_m$ and a scatter plot of $R_m$ versus $R_p$ are shown in Fig. 2. In the scatter plot and all subsequent analysis data with $R_m < 0.5$ MN were discarded, since $R_m > 1$ MN is anticipated in the present example and to avoid division by zero in the computation of $E_{pm}$ for Eq. (6). The linear regression fit of the remaining 70 data points in Fig. 2b confirms the visual impression that the data cloud is roughly distributed about a straight line, which, however, does not pass through the origin. As a consequence, the proportional bias model of Eqs. (2) and (3) is not entirely appropriate and instead of directly using $R_p$ from Eq. (1) as the predicted resistance, we use a site-specifically adjusted version $R_p' = 0.84R_p - 0.38$ based on the linear regression fit. This leads to $E[R_m] = E[R_p] = 1.40$ by taking the respective sample means and, $R_m = 1.16$ from Paikowsky [1], it yields $R_p' = 1.16$ by Eq. (4). Data further deliver $CV_{pm} = 0.32$ and $CV_{R_p} = 0.29$ as the sample coefficients of variation and, in combination with $CV_p = 0.34$ from Paikowsky [1], Eq. (5) gives $CV_p = 0.37$. Computing $E_{inp} = 1.00$ as the mean of all ratios $x_{in}R_m(x_{in}R_p)$ and $E_{pm} = 1.03$ as the mean of all ratios $x_{in}R_p(x_{in}R_m)$. Eq. (6) yields $\rho_{pm} = 0.88$. By Eqs. (9) and (10) this translates into estimation weights $w_p = 0.20$ and $w_m = 0.80$, further resulting in $CV_{pm} = 0.34$ from Eq. (11).

At the example site, single pile load test data for evaluation of $\rho_p$ through Eq. (13) is not available. We opt to apply Eq. (18) with an empirical value of $CV_p$ inferred from a series of load test results on pile groups at different sites of mainly $n_p = 9$ piles compiled by Jiang et al. [16, Table 2]. They report a maximum (most conservative) $CV_p = 0.24$ for freestanding pile groups in cohesionless soils. Without explicit information about the resistance prediction uncertainty methods involved in the load tests an exemplary value of $CV_p = 0.35$ is adopted [16, Table 1] as the resistance prediction uncertainty for single piles. With this, Eq. (18) delivers $\rho_p = 0.40$. Furthermore, partially based on results of Jiang et al. [16] and a survey of common practice/state-of-the-art, Paikowsky [1] recommends a target reliability of $\beta = 3$ for pile groups and a reduced $\beta = 2.33$ for single piles in redundant ($n_p \geq 5$) pile groups. Thus, applying Eq. (21) to a single pile with an exemplary value of $CV_p = CV_p = 0.35$ and $\beta = 2.33$ results in a design value of $\Phi = 0.55$. This leads to a pile group uncertainty $CV_p = 0.25$, obtained from inversion of Eq. (21) with $\beta = 3$ and $\Phi = 0.55$ (i.e., equal design based on pile and group reliabilities). In other words, the recommended reliability reduction from $\beta = 3$ to $\beta = 2.33$ between pile groups and redundant piles is equivalent to an assumed uncertainty reduction from a chosen $CV_p = 0.35$ for single piles to $CV_p = 0.25$ for pile groups. By Eq. (18) this leads to an estimate of $\rho_p = 0.44$ for $n_p = 9$, which is in reasonable agreement with $\rho_p = 0.40$ from above. In fact, Eq. (18) demonstrates that $\rho_p$ increases with $n_p$ to reach an asymptotic value of $\rho_p = CV_p/\sqrt{CV_p^2} = 0.50$, which is adopted in the sequel as a conservative upper bound for both predicted and monitored resistance estimation errors. The intermediate value of $\rho_p$ between zero and one also indicates that portions of spatial variability of estimation errors are contained within pile groups as well as between pile groups.

To continue the example, suppose that a pile group consists of 5 piles with monitoring on two piles (i.e., $n_p = 5$ and $n_m = 2$) and that the design load $Q_{des} = 5$ MN for the group with a target reliability of $\beta = 3$. Eqs. (15)–(17) and (21) immediately yield CV_{G0} = 0.16, CV_{G1} = 0.35, CV_{G2} = 0.27 and $\Phi = 0.52$. Using the fundamental LRDF design equation we arrive at $R_n = Q_{des}/(\Phi N_p) = 1.93$ MN for the nominal resistance of a single pile. The criteria for stopping pile advance are, thus, found as $R_p = W_p \rho_p R_p^1 + W_m \rho_m R_m = 0.19R_n + 0.93R_n - 0.09 = R_n$ for monitored piles and $R_p = \rho_p R_p^1 = 0.97R_n - 0.44 = R_n$ for unmonitored piles. Furthermore, we consider the scenario where one pile of the group was already driven to a resistance $R_n = R_p^1 = 2.6$ MN during the test pile program. Of the remaining four piles, one is to be monitored, such that $n_p = 5$, $n_m = 2$ and $n_0 = 1$ apply. Using the iterative procedure of Section 2.5 we arrive at $\Phi = 0.55$ and $R_n = 1.63$ MN. With this, the driving criteria of the remaining piles may be defined analogous to above. It may be observed that in spite of a rather small change in $\Phi$ the latter values of $R_n$ are significantly smaller than those obtained for $n_0 = 0$. This may be attributed to the fact that the presence of a rigid pile cap is assumed to perfectly redistribute excess loads from failed piles to still intact piles (e.g., stronger test piles). It is noted that if $R_n$ happens to be equal to $R_p$, for $n_0 = 0$, then the iteration process of Section 2.5 delivers identical results to those of the direct solution in Section 2.3.

4. Discussion of results

Fig. 3 graphically represents further results in terms of LRFD $\Phi$ as a function of the degree of monitoring $n_0/n_m$, which are based on the practical example in combination with the parameters given in each chart. The circles correspond to the hypothetical scenario of a single pile in a group ($n_p = 1$) and represent minimum values of $\Phi$. Asterisks are used for $n_p = 5$ and the continuous lines correspond to maximum values of $\Phi$ that are approached as $n_p$ becomes very large ($n_p \gg 1$). It may be consistently observed that $\Phi$ grows approximately linearly with $n_p/n_m$ between a minimum and a maximum value. These bounds correspond to a maximum group uncertainty $CV_p^2_{max} = CV_p^2[1/\rho_p + (1-1/n_p)]$ and a minimum group uncertainty $CV_p^2_{min} = CV_p^2(1/\rho_p + (1 - 1/n_p))$ which are obtained from Eqs. (15)–(17) by using $n_m/n_p = 0$ and 1, respectively. Fig. 3a shows that $\Phi$ is relatively constant with $n_m/n_p$. This may be attributed to the large value of $\rho_{pm} = 0.88$ resulting in an insubstantial uncertainty reduction through monitoring. Mathematically, this is reflected by $CV_p \approx CV_p$ and $CV_{p_{max}} \approx CV_{p_{min}}$. In other words, the high correlation between the estimation errors of predicted and monitored resistances observed in the practical example renders pile monitoring largely redundant with respect to prediction equations. As a consequence, the benefit of (additional) monitoring is limited and may not be worth the cost. However, this observation should not be arbitrarily generalized, since $\rho_{pm}$ in the practical example is inferred from depth profiles of only two piles, such that the ratios $R_{max}$ and $R_p/R_m$ may not contain the full site variability (i.e., $E_{np}$ and $E_{mp}$ in Eq. (6) would be larger with more data, thus decreasing $\rho_{pm}$). Smaller values of $\rho_{pm}$ also appear plausible in view of the fact that the random errors of prediction and monitoring methods may be attributed to quite different factors (e.g., hammer performance for predictions and strain sensor precision for monitoring). Moreover, $\rho_{pm}$ near unity would imply basically identical performance of prediction equations and monitoring approaches.

Fig. 3b illustrates the effect of a decrease in $\rho_{pm}$ to a hypothetical value of 0.1. While this does not affect $\Phi$ for $n_m/n_p = 0$ (no monitoring and, hence, $\rho_{pm}$ irrelevant), the reduced correlation leads to increasingly higher values of $\Phi$ as the degree of monitoring grows. In Fig. 3c prediction uncertainty $CV_p$ is hypothetically raised from 0.37 to 0.6. As to be expected, this leads to a lowering of $\Phi$, which is more pronounced for low degrees of monitoring and which becomes weaker as the influence of monitoring grows. Fig. 3d demonstrates the effect of decreasing spatial correlation $\rho_s$ from 0.5 to a hypothetical value of 0.1. For $n_p = 1, \rho_s$ is irrelevant and no changes are observed. For $n_p > 1, \Phi$ is seen to increase more strongly, which is due to the elevated independence of estimation errors between piles and a larger degree of uncertainty reduction after summing those errors into a pile group uncertainty. For the
Fig. 2. Pile driving data from piles 1 and 8 of the Dixie Highway project in Florida. (a) Depth profiles of PDA/CAPWAP monitored resistances $R_m$ and (b) scatter plot of all $R_m > 0.5$ MN versus $R_p$ from modified Gates equation (Eq. (1)) with linear regression fit.

Fig. 3. LRFD $\Phi$ from Eq. (21) for $\beta = 3$ as a function of degree of monitoring $n_m/n_p$ for the practical example in combination with the parameters indicated in each chart. Continuous lines are asymptotic maxima for $n_p >> 1$, asterisks are for $n_p = 5$ and circles are minima for hypothetical case of $n_p = 1$. 
preliminary empirical estimate of $\rho_0 = 0.5$ from pile group load tests and current design practice. Fig. 3a–c illustrates the convenient fact that $\Phi$ is quite close to its asymptotic maximum value for $n_p \geq 5$. This closeness increases for $\rho_0 > 0.5$ and means that in an initial design phase a conservative minimum resistance factor $\Phi_{\text{min}}$ (corresponding to $CV_{\text{g},\text{min}}$ for $n_p = 5$ and $\rho_0 = 0.5$) and a conservative maximum resistance factor $\Phi_{\text{max}}$ (corresponding to $CV_{\text{g},\text{min}}$ for $n_p = 5$ and $\rho_0 = 0.5$) may be determined to linearly approximate $\Phi = \Phi_{\text{min}} + (\Phi_{\text{max}} - \Phi_{\text{min}})n_p/n_p$. This simple relationship is independent of the total number $n_p$ of piles in a group and relates the degree of monitoring $n_m/n_p$ directly to a value of LRFD $\Phi$. Fig. 3a–c further illustrates that, for an inferred value of $\rho_0 = 0.5$, maximum values of $\Phi$ for full monitoring range between approximately 0.55 and 0.65. This agrees well with current design practice as reflected by an efficiency factor $\Phi/\Phi_0 = 0.56$ reported by Paikowsky [1, Table 20] for $\beta = 2.33$ (single pile). It is recalled once more that the present approach applies bias correction at the very beginning, such that resulting values of $\Phi$ are identical to respective efficiency factors ($\Phi_0 = 1$). The partial superiority of the present approach (larger $\Phi/\Phi_0$) may be attributed to the combination of predicted and monitored resistances and, as such, becomes more pronounced for smaller values of $CV_p$ and $\rho_{\text{pm}}$ (Fig. 3b and c).

The proposed site-specific calibration of pile resistance predictions from dynamic equations to those of dynamic measurements (monitoring) from a test pile program attempts to ensure appropriate bias and uncertainty parameters for a site. However, hammer performance, for example, may be highly variable during pile driving within a single site [4]. That is, a successful initial calibration from a test pile program may deteriorate in the course of a pile driving job. It is recommended, therefore, that the relationship between predicted and monitored resistances be accompanied and/or up-dated as data from monitored production piles become available. In addition, validation load tests as proposed by Zhang [5] and Liang and Yang [12] may serve to gain additional confidence in reliable pile group construction over the whole site.

5. Summary

Many national construction standards require reliability based design (e.g., [6]) to assure acceptable levels of reliabilities of engineered systems. Driven pile groups are a common type of deep foundation and probabilistic load parameters as well as target reliabilities are generally defined at the pile group level, i.e., not for single piles or components thereof. Pile driving criteria, however, are required for individual piles, which is currently achieved by using different target reliabilities at the pile and group levels [1]. In the present work we consider piles under axial loading and present an approach for defining pile driving criteria for individual piles, while satisfying target reliability at the group level. Total pile resistances (side plus tip components) are estimated from dynamic predictions (e.g., Gates equation) at all piles and from dynamic monitoring (e.g., PDA/CAPWAP) at an arbitrary number of monitored piles in a group. Resistance estimates from equations may strongly depend on variable properties of the driving equipment and are site-specifically calibrated to monitored resistances from test pile programs, for example. Estimates from dynamic equations and monitoring are then combined through best linear unbiased estimation (BLUE) to arrive at improved resistance estimates of monitored piles. The latter are again combined with resistance estimates from dynamic equations at unmonitored piles to yield an estimated pile group resistance with uncertainty.

Based on this, expressions for LRFD $\Phi$ and pile driving criteria are formulated as a function of prediction uncertainty $CV_p$, monitoring uncertainty $CV_{\text{g},\text{min}}$, correlation coefficient between estimation errors of prediction and monitoring methods $\rho_{\text{pm}}$ and correlation coefficient $\rho_0$ between estimation errors at different piles within a group. While $CV_p$ and $\rho_0$ are inferred from load test data bases, $CV_p$ and $\rho_{\text{pm}}$ are indirectly calibrated to monitored resistances at an exemplary site (using Davison criterion through the FHWA modified Gates equation and PDA/CAPWAP monitoring). Results in Fig. 3 show that for $\rho_0 \geq 0.5$ and five or more piles in a group ($n_p \geq 5$) $\Phi$ becomes relatively insensitive to $n_p$ and grows approximately linearly with the degree of monitoring $n_m/n_p$ between a minimum and a maximum value. Overall, and as to be expected, $\Phi$ decreases with increasing $CV_p$ and $CV_{\text{g},\text{min}}$ (larger uncertainties) as well as with $\rho_{\text{pm}}$ and $\rho_0$ (more data redundancy). In order to benefit from a significant increase in $\Phi$ due to monitoring both $CV_p$ and $\rho_{\text{pm}}$ have to be small. Besides these general results for $\Phi$, an iterative method for reliability based pile group design in the presence of previously driven piles in the group is also presented. It allows for the flexibility to decrease nominal pile resistances of the remaining piles, if previously driven piles in the group (e.g., from test pile program) are advanced to larger depths (resistances).

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