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2001 Chinese Phys. Lett. 18 1467

(http://iopscience.iop.org/0256-307X/18/11/313)

Hole Burning in the Fock Space: from Single to Several Holes *

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(Received 23 May 2001)

In our previous papers we have studied the production of a single hole in the photon number distribution of a field state [Phys. Lett. A 240 (1998) 277; 253 (1999) 123]. In this letter we extend the procedure for the controlled creation of an arbitrary number of holes.

PACS: 42. 50. Ar, 42. 50. Dv, 03. 65. Bz

Recently, it has been shown that a hole can be burned in the photon number distribution (PND) of certain states of the radiation field in a controlled way from an experimental viewpoint.[1,2] This study has been inspired by another effect, known as persistent hole burning in spectral lines of gases and solids.[3] The aim of this letter is to extend our previous analysis of the generation of a single hole to the controlled production of field states in a high-Q cavity possessing several holes in their PND located at desired positions.

In fact, by considering a superposition state interpolating between two arbitrary coherent states[4] with the same mean number of photons, that is, $|\alpha_1\rangle = |r\exp(\mathrm{i}\theta_1)\rangle$ and $|\alpha_1'\rangle = |r\exp(\mathrm{i}\theta_1')\rangle$, the state

$$|\psi(\xi_1,\phi_1)\rangle = \mathcal{N}_1\left(\sqrt{\xi_1}\,|\alpha_1\rangle + \mathrm{e}^{\mathrm{i}\phi_1}\sqrt{1-\xi_1}\,|\alpha_1'\rangle\right). \tag{1}$$

Here the normalization constant is given by

$$\mathcal{N}_1 = \left[1 + 2\sqrt{\xi_1(1-\xi_1)}\,\mathrm{Re}\left(\mathrm{e}^{\mathrm{i}\phi_1}\langle\alpha_1|\alpha_1'\rangle\right)\right]^{-1/2}. \tag{2}$$

One finds in the number state representation

$$|\psi(\xi_1,\phi_1)\rangle = \mathcal{N}_1\,\mathrm{e}^{-r^2/2}\sum_{n=0}^{\infty}\frac{r^n}{\sqrt{n!}}\mathrm{e}^{\mathrm{i}n\theta_1} \cdot\left(\sqrt{\xi_1} + \mathrm{e}^{\mathrm{i}(\phi_1+n\Delta\theta_1)}\sqrt{1-\xi_1}\right)|n\rangle, \tag{3}$$

where $\Delta\theta_1 = \theta_1' - \theta_1$. Then, by fixing

$$\xi_1 = 1/2\,,\quad \phi_1 = (1 - N_1/N_1')\pi,\quad \Delta\theta_1 = \pi/N_1', \tag{4}$$

N_1 and N_1' being integers, the state (1) becomes

$$|\psi_1\rangle = \mathcal{N}_1\,\mathrm{e}^{-r^2/2}\sum_{n=0}^{\infty}\frac{r^n}{\sqrt{2\,n!}}\mathrm{e}^{in\theta_1} \cdot\left(1 + \mathrm{e}^{\mathrm{i}\left(1+(n-N_1)/N_1'\right)\pi}\right)|n\rangle \tag{5}$$

which possesses holes in the PND, $P_n = |\langle n|\psi\rangle|^2$, for the values of n in the sequence $\{N_1, N_1+2N_1', N_1+4N_1', \cdots\}$; if N_1' is large compared with the $\langle\hat{n}\rangle \sim |\alpha|^2 = r^2$, only the hole at $n = N_1$ is significant since the others lie far away from the peak of the PND where it effectively vanishes.[1]

A scheme to produce the state (1) in a high-Q cavity, employing a method of "quantum state engineering"[5] through a dispersive atom–field interaction in the cavity,[6] has been proposed in Ref. [2]. This consists in preparing a Rydberg atom in the coherent superposition

$$\left|\psi_A^{(1)}(0)\right\rangle = c_g^{(1)}|g\rangle + c_e^{(1)}|e\rangle, \tag{6}$$

where $|g\rangle$ and $|e\rangle$ represent the ground ($n = 50$) and excited ($n = 51$) states of the atom, respectively, and $|c_g^{(1)}|^2 + |c_e^{(1)}|^2 = 1$, in a Ramsey zone before it enters the cavity at which the atom–field interaction takes place. The effective Hamiltonian describing the dispersive interaction within the cavity is given by[7]

$$\hat{H}_{\mathrm{int}} = \hbar\omega_{\mathrm{eff}}\left(|i\rangle\langle i| - |e\rangle\langle e|\right)\hat{a}^\dagger\hat{a}, \tag{7}$$

where $\omega_{\mathrm{eff}} = 2\boldsymbol{d}^2/\delta$ and $\boldsymbol{d}$ is the atomic dipole moment associated with the $|g\rangle \to |e\rangle$ (frequency ω_0) transition, $\delta = \omega - \omega_0$ is the detuning parameter and $\hat{a}^\dagger$ ($\hat{a}$) is the photon creation (annihilation) operator of the field mode with frequency ω. With the cavity frequency adjusted close to resonance for the transition $|e\rangle \to |i\rangle$ ($n = 52$), but distinct from ω_0, this dispersive interaction produces a phase shift φ in the cavity field when the atomic state is $|e\rangle$ but has no effect when the atom is in the state $|g\rangle$, that is

$$\left(c_g^{(1)}|g\rangle + c_e^{(1)}|e\rangle\right)|\psi_{\mathrm{F}}(0)\rangle \to c_g^{(1)}|g\rangle|\psi_{\mathrm{F}}(0)\rangle + c_e^{(1)}|e\rangle\exp[\mathrm{i}\varphi_1\hat{a}^\dagger\hat{a}]\,|\psi_{\mathrm{F}}(0)\rangle, \tag{8}$$

where $\varphi_1 = \omega_{\mathrm{eff}}\,\tau_1$, τ_1 being the time the atom takes to cross the cavity. If the field in the cavity is initially in a coherent state, $|\psi_{\mathrm{F}}(0)\rangle = |\alpha_1\rangle$, then the entangled

*Supported in part by CAPES, CNPq and MCT/PRONEX, Brazilian agencies.

atom–field state just after the cavity becomes

$$\left|\psi_{\rm AF}^{(1)}\right\rangle = c_g^{(1)}\left|g\right\rangle\left|\alpha_1\right\rangle + c_e^{(1)}\left|e\right\rangle\left|\alpha_1'\right\rangle, \tag{9}$$

where $\alpha_1' = \alpha_1 {\rm e}^{{\rm i}\varphi_1}$.

Now if the atom crosses a second Ramsey zone suffering a $\pi/2$ rotation in the atomic space, that is, $|g\rangle \to (|g\rangle - |e\rangle)/\sqrt{2}$, and $|e\rangle \to (|g\rangle + |e\rangle)/\sqrt{2}$, then the entangled atom–field state becomes $|g\rangle\,(c_g^{(1)}|\alpha_1\rangle + c_e^{(1)}|\alpha_1'\rangle)/\sqrt{2} + |e\rangle\,(-c_g^{(1)}|\alpha_1\rangle + c_e^{(1)}|\alpha_1'\rangle)/\sqrt{2}$. By detecting the atom in its ground state $|g\rangle$ the field inside the cavity is projected in the state

$$|\psi_1\rangle = \mathcal{N}_1(c_g^{(1)}|\alpha_1\rangle + c_e^{(1)}|\alpha_1'\rangle) \tag{10}$$

which reduces to the state (1) if one chooses $c_g^{(1)} = \sqrt{\xi_1}$ and $c_e^{(1)} = {\rm e}^{{\rm i}\phi_1}\sqrt{1-\xi_1}$. In other words, the atomic coefficients are transferred to the superposition state of the field in the cavity. Furthermore, by controlling the atomic velocity and the intensity of the first Ramsey zone, which determine the values of the parameters $c_g^{(1)}$, $c_e^{(1)}$ and φ_1, to match the conditions (4)

$$\varphi_1 = \frac{\pi}{N_1'}, \quad c_g^{(1)} = \frac{1}{\sqrt{2}}, \qquad c_e^{(1)} = \frac{1}{\sqrt{2}}\exp\left[{\rm i}(1-\frac{N_1}{N_1'})\pi\right], \tag{11}$$

the state (10) reduces to $|\psi_1\rangle$ and a hole is created in the PND of state (1) as described earlier.

The production of field states possessing more than one hole in their PND can be realized by performing a sequential selective detection of atoms crossing the cavity, in a way similar to that used to generate circular states.[8] We consider a second atom passing by the cavity after it has been prepared (in a Ramsey zone) in the normalized state

$$\left|\psi_A^{(2)}(0)\right\rangle = c_g^{(2)}|g\rangle + c_e^{(2)}|e\rangle, \tag{12}$$

and the field in the cavity being in the one-hole state (10). Owing to the dispersive interaction, just after the cavity the entangled atom–field state is

$$\begin{aligned}\left|\psi_{\rm AF}^{(2)}\right\rangle =& \mathcal{N}_1 c_g^{(2)}|g\rangle\left(c_g^{(1)}|\alpha_1\rangle + c_e^{(1)}|\alpha_1'\rangle\right)\\ &+ \mathcal{N}_1 c_e^{(2)}|e\rangle\left(c_g^{(1)}\left|{\rm e}^{i\varphi_2(\tau_2)}\alpha_1\right\rangle\right.\\ &\left.+c_e^{(1)}\left|{\rm e}^{{\rm i}\varphi_2(\tau_2)}\alpha_1'\right\rangle\right). \end{aligned}\tag{13}$$

Now, if the atom crosses a second Ramsey zone, suffering a $\pi/2$ pulse, before being detected in the ground state $|g\rangle$ then the field in the cavity is reduced to

$$\begin{aligned}|\psi_2\rangle =& \mathcal{N}_2\left[c_g^{(2)}\left(c_g^{(1)}|\alpha_1\rangle + c_e^{(1)}|\alpha_1'\rangle\right)\right.\\ &\left.+c_e^{(2)}\left(c_g^{(1)}|\alpha_2\rangle + c_e^{(1)}|\alpha_2'\rangle\right)\right], \end{aligned}\tag{14}$$

where $\alpha_2 = \alpha_1{\rm e}^{{\rm i}\varphi_2}$, $\alpha_2' = \alpha_1'{\rm e}^{{\rm i}\varphi_2} = \alpha_2{\rm e}^{{\rm i}\varphi_1}$ and

$$\begin{aligned}\mathcal{N}_2 =& \mathcal{N}_1\left\{1 + {\rm Re}\left[c_g^{(2)*}c_e^{(2)}\left(\langle\alpha_1|\alpha_2\rangle\right.\right.\right.\\ &\left.\left.\left.+c_g^{(1)*}c_e^{(1)}\langle\alpha_1|\alpha_2'\rangle + c_g^{(1)}c_e^{(1)*}\langle\alpha_1'|\alpha_2\rangle\right)\right]\right\}^{-1/2}. \end{aligned}\tag{15}$$

Since the phase difference between α_2' and α_2 coincides with that between α_1' and α_1, one notices immediately that the first hole burned is persistent, that is, not affected by the passage of the second atom also detected in $|g\rangle$, no matter how the values of the parameters are associated with it.

Now, the question posed is concerned with whether another hole can be burned by controlling the parameters of the second atom. Rewriting Eq. (14) in the form

$$\begin{aligned}|\psi_2\rangle =& \mathcal{N}_2\left[c_g^{(1)}\left(c_g^{(2)}|\alpha_1\rangle + c_e^{(2)}|\alpha_2\rangle\right)\right.\\ &\left.+c_e^{(1)}\left(c_g^{(2)}|\alpha_1'\rangle + c_e^{(2)}|\alpha_2'\rangle\right)\right] \end{aligned}\tag{16}$$

and noticing that the phase difference between α_2 and α_1 is the same as that between α_2' and α_1', namely φ_2, one sees that the analysis made for the first hole can be repeated for the passage of the second atom. Therefore, if the atomic velocity and the intensity of the first Ramsey zone are adjusted such that

$$\varphi_2 = \frac{\pi}{N_2'}, \quad c_g^{(2)} = \frac{1}{\sqrt{2}}, \qquad c_e^{(2)} = \frac{1}{\sqrt{2}}\exp\left[{\rm i}(1-\frac{N_2}{N_2'})\pi\right], \tag{17}$$

a second hole is burned in the PND of the state (14) at $n = N_2$.

This process can be continued; if a third atom crosses the system and is detected in its ground state, the field in the cavity becomes

$$\begin{aligned}|\psi_3\rangle =& \mathcal{N}_3\left\{c_g^{(3)}\left[c_g^{(2)}\left(c_g^{(1)}|\alpha_1\rangle + c_e^{(1)}|\alpha_1'\rangle\right)\right.\right.\\ &\left.+c_e^{(2)}\left(c_g^{(1)}|\alpha_2\rangle + c_e^{(1)}|\alpha_2'\rangle\right)\right]\\ &+ c_e^{(3)}\left[c_g^{(2)}\left(c_g^{(1)}|\alpha_3\rangle + c_e^{(1)}|\alpha_3'\rangle\right)\right.\\ &\left.\left.+ c_e^{(2)}\left(c_g^{(1)}|\alpha_4\rangle + c_e^{(1)}|\alpha_4'\rangle\right)\right]\right\}, \end{aligned}\tag{18}$$

where $\mathcal{N}_3$ is the normalization constant, $\alpha_3 = \alpha_1{\rm e}^{{\rm i}\varphi_3}$, $\alpha_3' = \alpha_1'{\rm e}^{{\rm i}\varphi_3}$, $\alpha_4 = \alpha_2{\rm e}^{{\rm i}\varphi_3}$ and $\alpha_4' = \alpha_2'{\rm e}^{{\rm i}\varphi_3}$. One sees immediately that the phase differences between α_3' and α_3 and between α_4' and α_4 are equal to $\Delta\theta_1$ and, therefore, the first hole is not affected by the passage of the third atom. Similarly, by rearranging the expressions inside the square brackets in this equation, putting in evidence the coefficients relative to the first atom, it is shown that the second hole is also preserved in the state generated by the passage of the third atom. On the other hand, writing Eq. (18) in the form

$$
\begin{aligned}
|\psi_3\rangle = \mathcal{N}_3 \Big\{ & c_g^{(2)} c_g^{(1)} \left(c_g^{(3)} |\alpha_1\rangle + c_e^{(3)} |\alpha_3\rangle \right) \\
& + c_g^{(2)} c_e^{(1)} \left(c_g^{(3)} |\alpha_1'\rangle + c_e^{(3)} |\alpha_3'\rangle \right) \\
& + c_e^{(2)} c_g^{(1)} \left(c_g^{(3)} |\alpha_2\rangle + c_e^{(3)} |\alpha_4\rangle \right) \\
& + c_e^{(2)} c_e^{(1)} \left(c_g^{(3)} |\alpha_2'\rangle + c_e^{(3)} |\alpha_4'\rangle \right) \Big\} ,
\end{aligned} \tag{19}
$$

it follows that a third hole is created at $n = N_3$ if the parameters related to the third atom are chosen as in Eq. (17) with N_3 and N_3' replacing N_2 and N_2'.

Naturally, one can continue with this analysis and then conclude that, by passing successively K Rydberg atoms prepared such that

$$
\begin{aligned}
& \varphi_j = \pi / N_j', \;\; c_g^{(j)} = 1/\sqrt{2} \, , \\
& c_e^{(j)} = \exp\left[\mathrm{i}(1 - N_j/N_j')\pi \right] / \sqrt{2}
\end{aligned} \tag{20}
$$

for $j = 1, \cdots, K$, through a high-Q cavity containing a field initially in the coherent state $\left| r\mathrm{e}^{\mathrm{i}\theta_1} \right\rangle$ and detecting all of them in the ground state, the field state

$$
\begin{aligned}
& |\psi_K(r, \theta_1; N_K, N_K' \cdots, N_1, N_1'))\rangle = \mathcal{N}_K \, \mathrm{e}^{-r^2/2} \\
& \cdot \sum_{n=0}^{\infty} \frac{r^n}{\sqrt{2^K \, n!}} \mathrm{e}^{\mathrm{i}n\theta_1} \prod_{j=1}^{K} \left(1 + \mathrm{e}^{\mathrm{i}\left(1 + (n - N_j)/N_j'\right)\pi} \right) |n\rangle \, ,
\end{aligned} \tag{21}
$$

where $\mathcal{N}_K$ is the normalization constant, is created. This state possesses holes in the PND for $n \in \{N_1, N_2, \cdots, N_K\}$ which are the only effective ones provided that we choose $N_1', \cdots, N_K'$ sufficiently large. Having states of a field mode possessing holes in the PND at controlled positions may be of interest to optical storage and communications. Finally, we should remark that the results obtained here are not restricted to the realm of quantum optics. They can be easily extended to all systems which can be modelled by a quantum linear oscillator, such as an ion trapped in a magneto-optical device. Another example is a mesoscopic Josephson junction interacting with a nonclassical microwave, in the external field approximation, and with the junction Hamiltonian approximated by a quadratic form; one hole states of such a junction have been described recently.[9]

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