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Fuzzy AHP Assessment of Water Management Plans

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Abstract There are two mainstreams when using the analytic hierarchy process (AHP). One is the standard applications of crisp distributive and ideal mode versions. The other is characterised by fuzzification of the AHP methodology and by attempts to better tackle inherently uncertain and imprecise decision processes with quantitative and qualitative data. The latter is characterised by different approaches to fuzzificating the decision problem; the way of conducting judgment and evaluating process; and finally, in synthesising the results and manipulating fuzzy numbers to devise priorities for the decision alternatives. This paper presents a fuzzy methodology for solving fully structured decision problems with criteria, sub-criteria and alternatives. It follows the logic of AHP in a simple and straightforward manner, efficiently aggregates criteria and sub-criteria into unique hierarchical level and applies a total integral method for comparing decision alternatives. The proposed methodology has been used for the assessment of water management plans in part of the Paraguacu River Basin in Brazil.

Keywords Multi-criteria analysis · Fuzzy extent method · Analytic hierarchy process (AHP)

1 Introduction

The performance evaluation of possible water management scenarios in a river basin is a multi-criteria decision making problem. In assessing the various interests of participating groups in a decision process it is necessary to clearly identify the overall goal and the hierarchically structured sets of criteria and sub-criteria that should be used in evaluating management scenarios as the decision alternatives. The problem that arises is that traditional multi-criteria

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methods (such as various mathematical programming methods) are not robust when dealing with limited experimental data, human judgments and the various metrics of decision variables. The main difficulties appear when quantitative measures should be combined with linguistic expressions and the decision makers attitudes toward risk need to be modelled appropriately.

The original crisp AHP method (Saaty 1980), and its extension fuzzy AHP (Laarhoven and Pedrycz 1983; Boender et al. 1989), proved to be an efficient tool for approaching these problems in water management (Fatti 1989; Ridgley 1993). The core of both versions of the method is the hierarchical structuring of the decision problem, followed by the systematic process of the synthesis of various judgments in order to derive priorities amongst criteria and subsequently the performance of alternatives. AHP uses pairwise comparisons of criteria and alternatives to form a reciprocal decision matrix, thus transforming qualitative data to crisp or fuzzy ratios. The eigenvector method is used to solve the reciprocal matrix and to determine the importance of criteria and the performance of alternatives with respect to criteria. The additive weighting method is used to calculate the utility of each alternative across criteria. In the case of fuzzy AHP, defuzzification is necessary at the end to obtain crisp weights and finally rank the alternatives.

The AHP and its variations have become a landmark in modern decision making due to several factors: (a) its ability to handle uncertain, imprecise and subjective data; (b) its robustness when solving practical ranking problems; (c) its methodological clearness and mathematical simplicity; and (d) its transparency to fuzzy logic and fuzzy sets theory (Tong and Bonissone 1984; Zimmermann 1987; Chen and Hwang 1992; Deng 1999). To solve multi-criteria problems involving qualitative data, Laarhoven and Pedrycz (1983) and Buckley (1985) extended Saaty's crisp AHP to deal with the decision maker's subjectivity in judgments by imbedding it into a fuzzy environment. A fuzzy version of the method was based on the use of triangular fuzzy numbers in pairwise comparisons in order to compute criteria weights and the overall utilities of alternatives, known as fuzzy utilities. In order to arrive at the final stage where alternatives are prioritised, the fuzzy utilities are required to be defuzzified and ranked. Some other fuzzy methods for prioritisation in AHP which are worth mentioning are, for example, those based on polyoptimisation as proposed by Wagenknecht and Hartmann (1983); fuzzy least squares by Xu (2000); or pseudo-inverse generalisation by Kwiesielewicz (1998). Although the latter three methods have gained a certain level of attention and are considered to be theoretically better, the fuzzy extent analysis method, as proposed in (Laarhoven and Pedrycz 1983) and (Buckley 1985), is more widely accepted in practice. This is probably due to its transparency and simplicity in handling uncertainties imbedded into decision making which includes quantitative, qualitative and 'grey' decision variables.

As highlighted by Deng (1999), the application of fuzzy AHP may produce unreliable results if: (a) an unbalanced 9-point scale is used; (b) the scale of fuzzification is not fully justified; and (c) an inappropriate defuzzification method is applied. Consequently, any application of fuzzy AHP requires considerable computations, careful handling of fuzzy operations and consistent interpretation of any results obtained.

The defuzzification itself is not a part of AHP, and the methods of deriving crisp weights for alternatives that enable their final ranking are subject to continuing research. When manipulating a fuzzy performance matrix (alternatives versus criteria) the most frequently used method is α-cut to obtain a confidence interval performance matrix. Following this is the incorporation of the optimism index of the decision maker in order to linearly transform intervals into crisp values. The final result, a crisp performance matrix, may thereafter additionally be analyzed to derive final ranks of alternatives. For example, the TOPSIS method (Hwang and Yoon 1981) is often used after the vector-matching process has been performed in order to normalise the values in the crisp performance matrix. This method defines that the best alternative should have the shortest Euclidean distance from the positive (maximum) ideal solution, and the farthest distance from the negative (minimum) ideal solution. In practice, the application of TOPSIS makes sense only if the number of alternatives is greater than three.

To keep the complex and unreliable process of comparing fuzzy utilities within reasonable limits, this paper presents an approach that: (1) completely follows a methodology of the fuzzy AHP method in parts of pairwise comparisons; (2) considers a case when some criteria split into sub-criteria and performs their mapping into a sub-criteria set; (3) uses fuzzy extent analysis and the additive weighting method (Hwang and Yoon 1981; Chen and Hwang 1992) in calculating utilities for alternatives across criteria and sub-criteria; and (4) applies the total integral value method (Liou and Wang 1992) to defuzzify utilities and to finally rank alternatives.

The fuzzy decision making (FDM) approach proposed here has been applied to the ranking of several long-term scenarios of water management in the Paraguacu river basin in Brazil (Srdjevic et al. 2002), and the results are presented in brief so as to illustrate computation procedure. Tests across the examples presented in pertinent literature (for example Deng 1999; Triantaphyllou and Lin 1996; Cheng 1996), reproduced most of the final results published by other authors, and proved our approach computationally stable and robust.

Proposed fuzzy AHP approach falls into wide framework of fuzzy sets and fuzzy theory applied to water resources. Several recently reported approaches are worth to be mentioned here. Karnib (2004) puts a focus on elaborating priority preorders of water resources projects through multi-criteria evaluation and fuzzy sets analysis. An interval fuzzy multiobjective programming method based on systems analysis was used in (Wang et al. 2006) to solve an integrated watershed management problem for the Lake Qionghai watershed in China. An application of fuzzy multi-objective function on reducing groundwater demand for aquaculture in land-subsidence areas presented in (Yang and Yu 2006) is based on comprising three single-objectives (reducing saltwater demand, reducing freshwater demand, and increasing the total fisheries gross profit) and coupling with a global optimization algorithm to find suitable aquaculture scenarios in the study area and provide the fisheries authorities with new references for revising the aquaculture structure. Singh et al. (2007) proposed an interactive fuzzy multi-objective LP approach on water quality management of a stretch of river Yamuna in India. The presented model will simulate the allocation of waste load efficiencies with satisfactory results which will indicate usefulness of the model in managing more complex river basins along with flexible policies of water management.

2 Basic Concepts

2.1 Fuzzy Sets, Norms and Extensions

The theory of fuzzy sets (Zadeh 1965) defines a fuzzy set A by degree of membership $\mu_A(x)$ over a universe of discourse X as:

$$\mu_A : X \rightarrow [0, 1] \tag{1}$$

Operations on fuzzy sets use connectives known as triangular norms T and S. T norms model the intersection operator in set theory, and S norms likewise model the union operator. Although the family of T and S norms is large, the min and max operators, as defined by Zadeh, are the most frequently used. For connecting fuzzy sets, the most commonly used are also the composition operators sup and inf. Respectively, the first is the supremum of its membership function over the universe of discourse, and the second is the infinum. Combinations of norms and composition operators enable operations on fuzzy sets, known as fuzzy operations.

Fuzzy arithmetic is made possible by Zadeh's extension principle which states that if f: $X \rightarrow Y$ is a function and A is a fuzzy set in X, then $f(A)$is defined as:

$$\mu_{f(A)}(y) = \sup_{x \in X, f(x)=y} \mu_A(x) \tag{2}$$

where: f: $X \rightarrow Y$, $y \in Y$. Based on the extension principle, it is possible to describe fuzzy arithmetic operations such as addition, subtraction, multiplication, division, inversion, logarithmisation or exponentiation (Triantaphyllou and Lin 1996; Bender and Simonovic 2000).

2.2 Positive Triangular Fuzzy Numbers

Positive triangular fuzzy numbers A are a special class of fuzzy number often expressed as $A=(a_1, a_2, a_3)$; where a_1, a_2, and a_3 are three real numbers satisfying $a_1>0$ and $a_1 \leq a_2 \leq a_3$. Any real number in interval $[a_1, a_3]$, is characterised with a grade of membership between 0 and 1. Its membership function $\mu_A(x)$ is piecewise continuous and linear, Fig. 1, and satisfies the following conditions:

1. $\mu_A(x)=0$, $\forall x \in (-\infty, a_1] \cup [a_3, \infty)$
2. $\mu_A(x)=1$, $x=a_2$
3. $\mu_A(x) = (x - a_1)/(a_2 - a_1)$, $\forall x \in [a_1, a_2]$
4. $\mu_A(x) = (a_3 - x)/(a_3 - a_2)$, $\forall x \in [a_2, a_3]$

The most probable value of fuzzy number A is modal value a_2. The lower and upper bounds a_1 and a_3 respectively, support the modal value and they illustrate its degree of fuzziness. The greater a_3-a_1 is, the fuzzier the degree is. When $a_3-a_1=0$, the value a_2 is not a fuzzy number and if $a_3-a_2=a_2-a_1$, the triangular fuzzy number A is symmetrical.

2.3 Methods of Defuzzification

For the given fuzzy number $A=(a_1, a_2, a_3)$, its crisp value may be obtained via various methods. The two methods used in the case study are:

- The centre of gravity: defuzzy$A = [(a_3 - a_1) + (a_2 - a_1)]/3 + a_1$

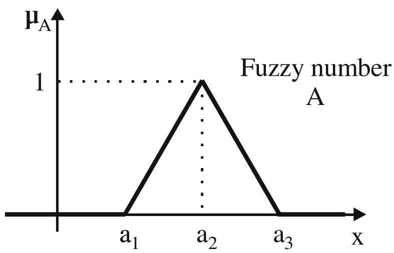

Fig. 1 Positive triangular fuzzy number

- The total integral value: defuzzy$A = (1/2)[\lambda a_3 + a_2 + (1-\lambda)a_1]$ (with $\lambda \in [0,1]$ being an optimism index).

2.4 The Value of Fuzzy Synthetic Extent

Let $X=\{x_1, x_2,..., x_n\}$ be an object set, and $U=\{u_1, u_2,..., u_m\}$ be a goal set. Fuzzy extent analysis (Chang 1996) can be performed with respect to each object for each goal respectively; and the result is m extent analysis values for each object given as:

$$\mu_i^1, \mu_i^2, \ldots, \mu_i^m, \quad i = 1, \ldots n.$$

All $\mu_i^j (i = 1, \ldots, n; j = 1, \ldots, m)$ are triangular fuzzy numbers representing the performance of the object x_i with regard to each goal u_j.

The value of fuzzy synthetic extent with respect to the ith object is defined as:

$$S_i = \sum_{j=1}^{m} \mu_i^j \otimes \left[\sum_{k=1}^{n} \sum_{l=1}^{m} \mu_k^l \right]^{-1}. \tag{3}$$

3 AHP Method

3.1 Crisp AHP

The decision problem structured in a 'crisp' (or 'non-fuzzy') AHP manner is to determine the weights of a given set of alternatives with respect to the specified overall goal by taking into consideration given sets of 'mediating' criteria, sub-criteria, sub sub-criteria and so forth. For the created hierarchy of the problem, an assessment of the mutual importance of elements should be performed at each level of the hierarchy. An assessment consists of pairwise comparisons of elements at a designated level with respect to the elements of the upper level. This procedure is repeated, in a downwards manner, for all levels. In the synthesis part of the process, simple matrix manipulations are performed with created judgmental matrices on order to obtain the performance ratings of alternatives.

Without becoming lost in generalities, an overall assumption can be made that the hierarchy of the decision problem consists only of a goal (G), a set of criteria C_j ($j=1, 2,..., M$) and a set of alternatives A_i ($i=1, 2,..., N$). This hierarchy may be called a two-level hierarchy, with levels counting from top to bottom as illustrated in Fig. 2.

AHP starts by performing a sequence of Mx$(M-1)/2$ pairwise comparisons of criteria with respect to a goal by using Saaty's original (crisp) 9-point scale, as defined in first two columns of the Table 1. In this manner a judgment matrix (Eq. 4) is created:

$$A = \begin{bmatrix} a_{11} & a_{12} & \ldots & a_{1M} \\ a_{21} & a_{22} & \ldots & a_{2M} \\ \ldots & \ldots & \ldots & \ldots \\ a_{M1} & a_{M2} & \ldots & a_{MM} \end{bmatrix} \tag{4}$$

with entries a_{ij} ($i,j=1, 2,..., M$) being crisp values given in the first column of Table 1, and where $a_{ij}=1$ for all $i=j$ ($i,j=1, 2,..., M$), and $a_{ij}=1/a_{ji}$. By normalizing values in each column followed by an averaging of normalised values in each row, a weighting vector

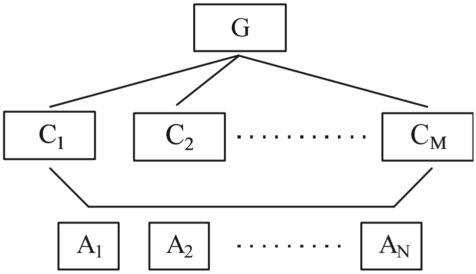

Fig. 2 Hierarchy of criteria and alternatives

(Eq. 5) is obtained with the elements w_1, w_2,..., w_M representing the relative importance of criteria with respect to a goal.

$$W = (w_1, w_2, \ldots, w_M), \sum_{i=1}^{M} w_i = 1. \tag{5}$$

Subsequently, pairwise comparisons of alternatives are performed at level 2 with respect to each criterion at level 1. A set of M matrices of size $N \times N$ is thus created. For each matrix a priority vector is computed by normalisation and by averaging in the same manner as before. Computed vectors represent the columns of the new matrix (Eq. 6). It should be observed that elements of jth vector are partial ratings of alternatives with respect to the jth criterion and they also add to 1.

$$X = \begin{array}{c} \begin{matrix} w_1 & w_1 & \ldots & w_M \end{matrix} \\ \begin{bmatrix} x_{11} & x_{12} & \ldots & X_{1M} \\ x_{21} & x_{22} & \ldots & x_{2M} \\ \ldots & \ldots & \ldots & \ldots \\ X_{N1} & x_{N2} & \ldots & x_{NM} \end{bmatrix} \end{array}. \tag{6}$$

Finally, priority vectors are multiplied by related criteria weights (ω_1, ω_2,...ω_M) to obtain a matrix (Eq. 7) which aggregates the performance ratings of all alternatives with respect to all criteria.

$$Z = \begin{bmatrix} w_1x_{11} & w_2x_{12} & \ldots & w_Mx_{1M} \\ w_1x_{21} & w_2x_{22} & \ldots & w_Mx_{2M} \\ \ldots & \ldots & \ldots & \ldots \\ w_1x_{N1} & w_2x_{N2} & \ldots & w_Mx_{NM} \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} & \ldots & z_{1M} \\ z_{21} & z_{22} & \ldots & z_{2M} \\ \ldots & \ldots & \ldots & \ldots \\ z_{N1} & z_{N2} & \ldots & z_{NM} \end{bmatrix} \tag{7}$$

Table 1 Original and fuzzified Saaty's scale for pairwise comparisons

Saaty's crisp values (x)	Judgment definition	Fuzzified Saaty's values
1	Equal importance	$(1, 1, 1+\delta)$
3	Week dominance	$(3-\delta, 3, 3+\delta)$
5	Strong dominance	$(5-\delta, 5, 5+\delta)$
7	Demonstrated dominance	$(7-\delta, 7, 7+\delta)$
9	Absolute dominance	$(9-\delta, 9, 9)$
2, 4, 6, 8	Intermediate values	$(x-1, x, x+1)$, x=2, 4, 6, 8

δ is fuzzy distance ($0.5 \le \delta \le 2$).

The summation of the elements in each row of matrix Z gives the final result (Eq. 8): the weights for alternatives at the fingertips of the hierarchy with respect to a goal at the top of the hierarchy.

$$w_i = \sum_{j=1}^{M} z_{ij}, i = 1,\ 2, \ldots, N. \tag{8}$$

To summarise, the final step of the AHP method deals with the construction of an $N \times M$ matrix Z given by Eq. 7, where N is the number of alternatives, and M is the number of criteria. This matrix is usually called a performance matrix and it serves as the basis for a final assessment and the rating of the alternatives. In crisp AHP it is done by Eq. 8, and in fuzzy AHP it is done in a way that is to be described later.

The described prioritisation method, based on the normalisation and averaging of columns and of rows of reciprocal matrices, is commonly used in practice. For the sake of completeness, other prioritisation methods based on eigenvector and eigenvalue may also be used if reciprocal matrices are significantly inconsistent. An example of this is given in a discussion by the author of the AHP in (Saaty 1990).

3.2 Fuzzy AHP

Different fuzzy based multi-criteria analysis models have been developed, including many that more or less follow the AHP philosophy (for instance Triantaphyllou and Lin 1996; Raju and Pillai 1999; Arslan and Khisty 2006). The most usual are those that completely imitate standard AHP and its principles of manipulating priority vectors derived from judgment matrices in assessing the elements of the hierarchy, and consequently apply fuzzy arithmetic throughout the process.

The mainstream models exploit the concept of the fuzzy extent analysis after the AHP formulation of the problem is imbedded into a fuzzy environment. The prerequisite for this concept is to fuzzify a 9-point scale and to continue assessments in a fuzzy manner until the final synthesis has to be performed. Some authors (such as Cheng 1996, or Deng 1999) use only odd integers 1, 3, 5, 7 and 9 to express the decision maker's subjective measure of dominance of one element over the other. In fuzzification by triangular fuzzy equivalents, the distance of 2 is commonly used; on boundaries, (1,1,1) or (1,1,3) is used for 1, and (7,9,9) or (7,9,11) is used for 9. Our analyses show that the most consistent results can be expected if Table 1 is used with a fuzzy distance of 2 for odds (3, 5, 7), and a fuzzy distance of 1 for pairs (2, 4, 6), with (1,1,3) and (7,9,9) at the boundaries.

3.3 Fuzzy Extent Analysis and Synthesis of Results

After the hierarchy of the multi-criteria problem has been created by using triangular fuzzy numbers and related membership functions, as defined in Table 1, the ranking procedure starts with the determining of the importance of criteria with respect to the goal. A fuzzy reciprocal judgment matrix for criteria importance is transformed into the triangular fuzzy weights of criteria via fuzzy extent analysis (Eq. 1). The same is repeated for alternatives with respect to criteria, and final the synthesis of all weights is performed by the additive method as in crisp AHP. The difference with respect to the crisp method is that Saaty's scale is fuzzified, all operations are fuzzy with triangular fuzzy numbers, and the final ranking of alternatives is performed after the defuzzification. The defuzzification can be performed differently (Deng 1999; Triantaphyllou and Lin 1996; Zhu et al. 1999) and, likewise, the final results may differ.

4 The FDM Approach

The fuzzy decision making (FDM) approach is proposed here. It is inspired by crisp AHP and it comprises of several principles and procedures that pave the way for consequent analysis and the solving of a hierarchically structured decision problem. FDM involves qualitative assessments in fuzzy framework and it is based on the following premises:

1. Crisp AHP is fuzzified preserving its crisp logic and a method of manipulating the priority vectors.
2. The full range of Saaty's evaluation scale is fuzzified, not only are odd positive integer entries; triangular fuzzy numbers are used.
3. An aggregation principle is implemented when manipulating criteria that split into sub-criteria. The criteria and sub-criteria levels aggregate into a unique level.
4. Fuzzy extent analysis is applied in all instances.
5. The total integral value method is used for defuzzification and the final ranking of alternatives.

4.1 Fuzzifying Judgment Scale

Fuzzy numbers are intuitively easy to use when expressing the decision maker's qualitative assessments. To facilitate the making of pairwise comparisons in fuzzy AHP application, Saaty's original 9-point scale may be fuzzified as shown in the last two columns of Table 1. Membership functions for $\widetilde{1} \leq \widetilde{x} \leq \widetilde{9}$ are assumed to be symmetrically triangular, different for an internal pair and odd integers and adjusted for edge values along the scale. Note that pair fuzzy numbers $\widetilde{2}, \widetilde{4}, \widetilde{6}$, and $\widetilde{8}$ are fuzzified with $\delta=1$ due to their intermediate judgment positions within the scale, and that edge fuzzy numbers $\widetilde{1}$ and $\widetilde{9}$ are defined to reflect a real decision situation. According to the judgment definitions given in the second column of Table 1, the fuzzy distance for internal odd integers should be only within the interval $0.5 \leq \delta \leq 2$.

4.2 The Aggregation Principle

The set of M criteria: C_1, C_2,..., C_M. has been given. Each criterion may be decomposed into several sub-criteria. If k_j is the number of sub criteria related to the jth criterion, the total number of sub-criteria is:

$$K = \sum_{j=1}^{M} k_j. \tag{9}$$

The given criterion does not necessarily split into sub-criteria; in such a case its k_j is equal to 1 and criterion counts as sub-criterion as well. This is important for understanding the aggregation process of judgments made at two consecutive hierarchical levels, where criteria and sub-criteria are located. Here criteria and sub-criteria are aggregated by shifting criteria at the sub-criteria level, assuming that after that shift the whole criteria level does not exist anymore. It is equivalent to the situation that each criterion has at least one sub-criterion, namely itself. The aggregation principle is illustrated in Fig. 3.

4.3 Evaluating Criteria

The ranking procedure starts with the determination of the importance of criteria with respect to the goal. By using a fuzzified scale, a fuzzy reciprocal judgment matrix for criteria is determined as:

$$A = \begin{bmatrix} \widetilde{a}_{11} & \widetilde{a}_{12} & \dots & \widetilde{a}_{1M} \\ \widetilde{a}_{21} & \widetilde{a}_{22} & \dots & \widetilde{a}_{2M} \\ \dots & \dots & \dots & \dots \\ \widetilde{a}_{M1} & \widetilde{a}_{M2} & \dots & \widetilde{a}_{MM} \end{bmatrix} \tag{10}$$

where $\widetilde{a}_{ij} = 1$ for all $i=j$ $(i, j=1, 2,..., M)$, and $\widetilde{a}_{ij} = 1/\widetilde{a}_{ji}$.

By applying the fuzzy synthetic extent (Eq. 3), corresponding weights of criteria can be determined as:

$$w_i = \sum_{j=1}^{M} \widetilde{a}_{ij} \otimes \left[\sum_{k=1}^{M} \sum_{l=1}^{M} \widetilde{a}_{kl} \right]^{-1}, \quad i = 1, \dots, M. \tag{11}$$

All w_i, $i=1,..., M$, are normalised fuzzy numbers with medium values equaling 1. It should be noted that fuzzy extent (Eq. 11) could be defined as the result of fuzzy arithmetic, or by using the extension principle. The second is slightly more difficult, but would lead to reduced uncertainty.

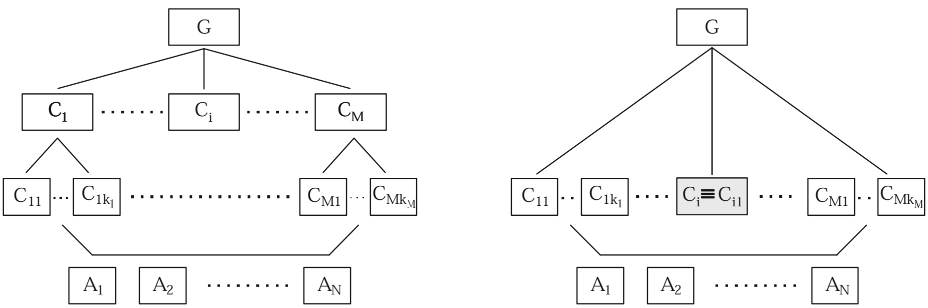

Fig. 3 Aggregation of criteria and sub criteria levels

4.4 Evaluating Sub-criteria

For the given criterion C_j, which splits into k_j sub-criteria, it is necessary to determine the relative importance of the sub-criteria with respect to this criterion. After that the fuzzy judgment matrix can be determined as:

$$A_j = \begin{bmatrix} \widetilde{a}_{11} & \widetilde{a}_{12} & \dots & \widetilde{a}_{1k_j} \\ \widetilde{a}_{21} & \widetilde{a}_{22} & \dots & \widetilde{a}_{2k_j} \\ \dots & \dots & \dots & \dots \\ \widetilde{a}_{k_j1} & \widetilde{a}_{k_j2} & \dots & \widetilde{a}_{k_jk_j} \end{bmatrix} - \tag{12}$$

The weights of sub-criteria with respect to given criterion are obtained again as fuzzy extents.

Final sub-criteria weights are derived through the aggregation of the weights at two consecutive levels. Multiplying sub-criteria weights by respective criterion weight (Eq. 11) gives:

$$w_j^p = \left(\sum_{l=1}^{k_j} \widetilde{a}_{il} \otimes \left[\sum_{i=1}^{k_j} \sum_{l=1}^{k_j} \widetilde{a}_{il} \right]^{-1} \right) \otimes w_j, \quad j = 1, \dots, M, p = 1, \dots k_j \tag{13}$$

where w_j^p are the aggregated fuzzy weights of sub-criteria. They are entries of the weight vector 14 with the total length K (cf. Eq. 9).

$$W = \left(w_1^1, w_1^2, \dots, w_1^{k_1}, w_2^1, w_2^2, \dots w_2^{k_2}, \dots, w_j^1, w_j^2, \dots w_j^{k_j}, \dots, w_M^1, w_M^2, \dots, w_M^{k_M} \right). \tag{14}$$

For simplicity, entries of vector 14 can be rewritten (by using letter ω instead of letter w) and renumbered to obtain Eq. 15 (again cf. Eq. 9).

$$W = (\omega_1, \omega_2, ..., \omega_K). \tag{15}$$

4.5 Evaluating Alternatives

The provided N alternatives are pairwise compared with respect to each of the K sub-criteria. After obtaining K fuzzy judgment matrices of type 16, the fuzzy extent 17 produces the decision matrix 18.

$$W_k = \begin{bmatrix} \widetilde{a}_{11} & \widetilde{a}_{12} & \dots & \widetilde{a}_{1N} \\ \widetilde{a}_{21} & \widetilde{a}_{22} & \dots & \widetilde{a}_{2N} \\ \dots & \dots & \dots & \dots \\ \widetilde{a}_{N1} & \widetilde{a}_{N2} & \dots & \widetilde{a}_{NN} \end{bmatrix}, k = 1, \dots, K \tag{16}$$

$$x_{ij} = \sum_{k=1}^{K} \widetilde{a}_{ik} \otimes \left[\sum_{l=1}^{N} \sum_{m=1}^{K} \widetilde{a}_{lm} \right]^{-1}, i = 1, \dots, N; j = 1, \dots, K \tag{17}$$

$$X = \begin{bmatrix} x_{11} & x_{12} & \dots & x_{1K} \\ x_{21} & x_{22} & \dots & x_{2K} \\ \dots & \dots & \dots & \dots \\ x_{N1} & x_{N2} & \dots & x_{NK} \end{bmatrix} \tag{18}$$

In the decision matrix X, x_{ij} represents the resultant fuzzy performance assessment of the alternative A_i (i=1, 2,..., N) with respect to the jth sub-criterion (j=1, 2,..., K).

4.6 Performance Matrix

As proposed by Deng (1999), an overall performance of each alternative across all sub-criteria may be represented by the fuzzy performance matrix 19.

$$Z = \begin{bmatrix} x_{11} \otimes \omega_1 & x_{12} \otimes \omega_2 & \ldots & x_{1K} \otimes \omega_K \\ x_{21} \otimes \omega_1 & x_{22} \otimes \omega_2 & \ldots & x_{2K} \otimes \omega_K \\ \ldots & \ldots & \ldots & \ldots \\ x_{N1} \otimes \omega_1 & x_{N2} \otimes \omega_2 & \ldots & x_{NK} \otimes \omega_K \end{bmatrix}. \tag{19}$$

It is obtained by multiplying the entries of the weighting vector 15 by the related column values of the decision matrix 18 and by applying fuzzy interval arithmetic. Recall that if a certain criterion does not split, it is considered as its own copy within the set of sub-criteria, and the value of its weight with respect to the goal is preserved.

4.7 Final Assessments and Synthesis

Several methods have been proposed to aggregate the decision maker's assessments. The most commonly used are the mean, median, max, min and mixed operators (Buckley 1985). Additive synthesis has been assumed here and the final alternative performance weights with respect to overall goal are calculated by the summation of elements in the rows of the performance matrix 19 to obtain Eq. 20.

$$F_i = \sum_{j=1}^{K} x_{ij} \otimes \omega_j \qquad i = 1, 2 \ldots, N. \tag{20}$$

To finally rank the alternatives, the prioritisation of aggregated assessments is required. Since each F_i is a triangular fuzzy number, it is necessary to apply the method of ranking triangular fuzzy numbers. There are several methods that are able to do this; such as the centre of gravity method, the dominance measure method, the α-cut with interval synthesis method, and the total integral value method. The last, the total integral value method (Liou and Wang 1992), is considered to be a good choice for performing the task efficiently and, therefore, has been proposed within this methodology.

For the given triangular fuzzy number A=(a_1, a_2, a_3), the total integral value is defined as:

$$I_T^\lambda(A) = (1/2)[\lambda a_3 + a_2 + (1 - \lambda)a_1], \ \ \lambda \in [0, 1]. \tag{21}$$

In Eq. 21, λ represents an optimism index which expresses the decision maker's attitude toward risk. A larger value of λ indicates a higher degree of optimism. In practical applications, values 0, 0.5 and 1 are used respectively to represent the pessimistic, moderate and optimistic views of the decision maker.

For given fuzzy numbers A and B it is said that if $I_T^\lambda(A) < I_T^\lambda(B)$, then $A \prec B$; if $I_T^\lambda(A) = I_T^\lambda(B)$ then A=B; and if $I_T^\lambda(A) > I_T^\lambda(B)$, then $A \succ B$.

The final ranking of alternatives means to adopt certain level λ of optimism of the decision-maker, then to apply Eq. 21 on fuzzy numbers Eq. 20, and finally to rank alternatives regarding obtained values for $I_T^\lambda(F_i), i = 1, \ldots N$.

5 An Example of the Application of FDM in Water Management

5.1 About the Decision Problem

A three-level hierarchy was created to test the proposed FDM approach and to verify its applicability in further developments. The overall goal has been stated as selecting the best long-term water management scenario for the Jacuipe River Basin (JRB), situated in the northern section of the Paraguacu River Basin (PRB) in Brazil. Three management scenarios were used as decision alternatives to be evaluated by means of 5 criteria which were split into a total of 24 sub-criteria.

For a better understanding of the decision framework it should be said that the PRB covers over 55,000 km^2. Within the basin there are 84 municipalities with nearly 2 million inhabitants. The prevailing climatic conditions are semi-arid with frequent droughts and the available waters of acceptable quality are predominantly superficial; groundwater is generally of a high salinity due to the existence of huge crystalline structures throughout the basin. There are more than 15 surface reservoirs acting as the major conservation facilities within basin. However, only three are important for global management within the northern part of the basin. In an upstream order the reservoirs are: Pedra do Cavalo ($5{,}000\times10^6$ m^3), Sao Jose de Jacuipe (355×10^6 m^3) and Franca (24×10^6 m^3). Primary water uses within the analysed part of the basin are: drinking water for both humans and animals, industrial supply and agricultural irrigation. Secondary uses are tourism, recreation, and ecological finalities.

Conflicts in water uses within the Jacuipe River Basin are evidenced as follows: (a) the upper basin – mechanised irrigation versus mineral research versus agricultural activities; (b) the medium and lower basin – irrigation versus water supply versus wastewater disposal and treatment; (c) the Sao Jose de Jacuipe dam – salinisation versus human water supply (drinking water) versus irrigation; and (d) the Pedra do Cavalo dam – human water supply (drinking water) versus hydropower production (once electric power facilities have been installed).

Management scenarios, used as decision alternatives, represent various conflicts of interest between groups in the basin. These are commonly recognised as different attitudes of the community in providing means and quantities of water supplies according to specified priorities (Srdjevic et al. 2002). To simplify presentation, only three strategic master plans and related management scenarios have been used in this case study. There are more scenarios, however they are considered to be dominated by the three which have been selected. The three management plans consider both present (2005) and future (2040) water supplies and the demands which are placed upon the JRB. Available water quantities are estimated according to two distinct aspects: (a) the 90% guaranteed discharge in two more upstream sub-basins and (b) stored water in reservoirs. The major uses of water within the basin are considered to be human supply, animal supply and irrigation. In addition, and as established by law, 20% of the available water should be retained in the riverbed for ecological and riparian needs.

Criteria and related sub-criteria are defined in broad sense to include political, economical, social, environmental and technical aspects of water management within the basin. In this way it was possible to 'simulate', by FDM methodology, the decision framework in which various interest groups at the basin, sub-basin and local levels participate through the JRB Water Committee in formulating criteria and sub-criteria. Concurrently responsible state agencies define alternative plans and present them properly to the decision making entities whatever they are; amongst others this includes representatives of the communities within the area, governmental agencies, the government itself and shareholders. However, the criteria and sub-criteria used here to demonstrate

FDM methodology will be under the continuous scrutiny of the community and will probably become withered under the real decision making process as described in (Srdjevic et al. 2002).

5.2 Hierarchy of the Decision Problem

The decision hierarchy is defined as follows:

1. Goal: to identify the best management plan (the economical, social and ambient related benefits for all)
2. Criteria (Level 1) and sub-criteria (Level 2):
   - A. Political impacts
     - A1: State and basin agencies and organisations
     - A2: In-basin water committees
     - A3: Human population in cities and villages
     - A4: Stakeholders
     - A5: Producers (agricultural and industrial)
     - A6: Local leaders (such as city majors)
   - B. Economical issues
     - B1: Implementing an economical process
     - B2: Reliability of economical parameters
     - B3: Costs (investment, operations and maintenance)
     - B4: Benefits (direct, indirect)
   - C. Social issues
     - C1: Infrastructure
     - C2: Demographic changes and migration
     - C3: Health care issues
     - C4: Working conditions
   - D. Environment & ambience
     - D1: Distribution of pleasant resorts
     - D2: Preserving cultural values
     - D3: Conditions for water conservation
     - D4: Accessing objects and facilities
     - D5: Protecting waters (water quality)
     - D6: Sanitary conditions
   - E. Technical criterion
     - E1: Spatial distribution of projects
     - E2: Technical conditions of projects
     - E3: Technologies involved (clean and dirty)
     - E4: Eligibility for technical improvements
3. Management plans (Level 3):

   Plan 1: Demands related to human supply and animal supply should be fully satisfied in the future at present level needs. The remaining waters should be used giving priority to irrigation according to future needs. In the case of any surplus waters, ecological demands should be satisfied.

   Plan 2: Priority should be given to attending to the demands of both human and animal supply, followed by irrigation demands, all according to future needs. Once more, in case of available water surplus, ecological demands should be satisfied.

Plan 3: This alternative considers fulfilling the necessities of human and animal supplies as the major priority; firstly according to future necessity values and then followed by ecological demands. Only in the case of available water surplus, should irrigation demands be satisfied.

5.3 Assessment, Synthesis and Ranking

To determine the relative importance of the evaluation criteria A–E, they were pairwise compared with respect to the goal by using the fuzzified scale given in Table 1. Linguistically expressed preferences among criteria have been used to create a judgment matrix A as given by Eq. 10.

$$A = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} \tilde{1} & \tilde{3}^{-1} & \tilde{2} & \tilde{3} & \tilde{4} \\ \tilde{3} & \tilde{1} & \tilde{2} & \tilde{3} & \tilde{5} \\ \tilde{2}^{-1} & \tilde{3}^{-1} & \tilde{1} & \tilde{1} & \tilde{3} \\ \tilde{3}^{-1} & \tilde{3}^{-1} & \tilde{1} & \tilde{1} & \tilde{4} \\ \tilde{4}^{-1} & \tilde{5}^{-1} & \tilde{3}^{-1} & \tilde{4}^{-1} & \tilde{1} \end{bmatrix} \end{array}.$$

Note that all entries in the matrix are fuzzy numbers from Table 1, each element on the main diagonal is a fuzzy number (1,1,3), and that entries in the upper and lower matrix triangles are reciprocals.

The weighting vector w of criteria matrix A was determined by applying Eq. 11 and fuzzy extent 3. Each entry of this vector is the sum of elements in the related row of matrix A, divided by the sum of all its elements. For example:

$$w_1 = \frac{\tilde{1}+\tilde{3}^{-1}+\tilde{2}+\tilde{3}+\tilde{4}}{\tilde{1}+\tilde{3}^{-1}+\tilde{2}+\tilde{3}+\tilde{4}+\tilde{3}+\tilde{1}+\tilde{2}+\tilde{3}+\tilde{5}+\tilde{2}^{-1}+\tilde{3}^{-1}+\tilde{1}+\tilde{1}+\tilde{3}+\tilde{3}^{-1}+\tilde{3}^{-1}+\tilde{1}+\tilde{1}+\tilde{4}+\tilde{4}^{-1}+\tilde{5}^{-1}+\tilde{3}^{-1}+\tilde{4}^{-1}+\tilde{1}}$$
$$= (0.090, 0.265, 0.728)$$

$$w = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \end{bmatrix} = \begin{bmatrix} (0.090, 0.265, 0.728) \\ (0.101, 0.359, 0.985) \\ (0.053, 0.154, 0.557) \\ (0.069, 0.171, 0.471) \\ (0.025, 0.052, 0.214) \end{bmatrix}.$$

In the next step, through the use of fuzzy pairwise comparisons, the judgment matrices 12 for sub-criteria related to respective criteria were obtained. Related sub-criteria weighting vectors were calculated as defined by Eqs. 11 and 3.

$$A_A = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ A_6 \end{array} \begin{array}{c} \begin{array}{cccccc} A_1 & A_2 & A_3 & A_4 & A_5 & A_6 \end{array} \\ \begin{bmatrix} \tilde{1} & \tilde{3} & \tilde{4} & \tilde{3}^{-1} & \tilde{2}^{-1} & \tilde{2} \\ \tilde{3}^{-1} & \tilde{1} & \tilde{2} & \tilde{2}^{-1} & \tilde{3}^{-1} & \tilde{1} \\ \tilde{4}^{-1} & \tilde{2}^{-1} & \tilde{1} & \tilde{3}^{-1} & \tilde{3}^{-1} & \tilde{1} \\ \tilde{3} & \tilde{2} & \tilde{3} & \tilde{1} & \tilde{2} & \tilde{2} \\ \tilde{2} & \tilde{3} & \tilde{3} & \tilde{2}^{-1} & \tilde{1} & \tilde{2} \\ \tilde{3}^{-1} & \tilde{1} & \tilde{1} & \tilde{2}^{-1} & \tilde{2}^{-1} & \tilde{1} \end{bmatrix} \end{array} \Rightarrow w_A = \begin{bmatrix} w_{11} \\ w_{12} \\ w_{13} \\ w_{14} \\ w_{15} \\ w_{16} \end{bmatrix} = \begin{bmatrix} (0.073, 0.226, 0.662) \\ (0.042, 0.108, 0.441) \\ (0.033, 0.071, 0.343) \\ (0.067, 0.271, 0.809) \\ (0.060, 0.219, 0.735) \\ (0.30, 0.104, 0.294) \end{bmatrix}$$

$$A_B = \begin{array}{c} \\ B_1 \\ B_2 \\ B_3 \\ B_4 \end{array} \begin{array}{c} \begin{array}{cccc} B_1 & B_2 & B_3 & B_4 \end{array} \\ \begin{bmatrix} \tilde{1} & \tilde{2} & \tilde{4} & \tilde{3} \\ \tilde{2}^{-1} & \tilde{1} & \tilde{2} & \tilde{2} \\ \tilde{4}^{-1} & \tilde{2}^{-1} & \tilde{1} & \tilde{1} \\ \tilde{3}^{-1} & \tilde{2}^{-1} & \tilde{1} & \tilde{1} \end{bmatrix} \end{array} \Rightarrow w_B = \begin{bmatrix} w_{21} \\ w_{22} \\ w_{23} \\ w_{24} \end{bmatrix} = \begin{bmatrix} (0.153, 0.474, 1.165) \\ (0.085, 0.261, 0.728) \\ (0.064, 0.130, 0.534) \\ (0.047, 0.134, 0.437) \end{bmatrix}$$

$$A_C = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \end{array} \\ \begin{bmatrix} \tilde{1} & \tilde{5} & \tilde{2} & \tilde{3} \\ \tilde{5}^{-1} & \tilde{1} & \tilde{5}^{-1} & \tilde{4}^{-1} \\ \tilde{2}^{-1} & \tilde{5} & \tilde{1} & \tilde{2} \\ \tilde{3}^{-1} & \tilde{4} & \tilde{2}^{-1} & \tilde{1} \end{bmatrix} \end{array} \Rightarrow w_C = \begin{bmatrix} w_{31} \\ w_{32} \\ w_{33} \\ w_{34} \end{bmatrix} = \begin{bmatrix} (0.130, 0.408, 1.037) \\ (0.032, 0.061, 0.231) \\ (0.116, 0.315, 0.807) \\ (0.099, 0.216, 0.576) \end{bmatrix}$$

$$A_D = \begin{array}{c} \\ D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \end{array} \begin{array}{c} \begin{array}{cccccc} D_1 & D_2 & D_3 & D_4 & D_5 & D_6 \end{array} \\ \begin{bmatrix} \tilde{1} & \tilde{3} & \tilde{2} & \tilde{3} & \tilde{2}^{-1} & \tilde{2}^{-1} \\ \tilde{3}^{-1} & \tilde{1} & \tilde{1} & \tilde{2}^{-1} & \tilde{3}^{-1} & \tilde{3}^{-1} \\ \tilde{2}^{-1} & \tilde{1} & \tilde{1} & \tilde{2} & \tilde{1} & \tilde{2} \\ \tilde{3}^{-1} & \tilde{2} & \tilde{2}^{-1} & \tilde{1} & \tilde{3}^{-1} & \tilde{2}^{-1} \\ \tilde{2} & \tilde{3} & \tilde{1} & \tilde{3} & \tilde{1} & \tilde{2} \\ \tilde{2} & \tilde{3} & \tilde{2}^{-1} & \tilde{2} & \tilde{2}^{-1} & \tilde{1} \end{bmatrix} \end{array} \Rightarrow w_D = \begin{bmatrix} w_{41} \\ w_{42} \\ w_{43} \\ w_{44} \\ w_{45} \\ w_{46} \end{bmatrix} = \begin{bmatrix} (0.053, 0.214, 0.711) \\ (0.033, 0.075, 0.395) \\ (0.053, 0.161, 0.553) \\ (0.035, 0.100, 0.395) \\ (0.061, 0.257, 0.789) \\ (0.053, 0.193, 0.632) \end{bmatrix}$$

$$A_E = \begin{array}{c} \\ E_1 \\ E_2 \\ E_3 \\ E_4 \end{array} \begin{array}{c} \begin{array}{cccc} E_1 & E_2 & E_3 & E_4 \end{array} \\ \begin{bmatrix} \tilde{1} & \tilde{3} & \tilde{3} & \tilde{5} \\ \tilde{3}^{-1} & \tilde{1} & \tilde{2} & \tilde{2} \\ \tilde{3}^{-1} & \tilde{2}^{-1} & \tilde{1} & \tilde{4} \\ \tilde{5}^{-1} & \tilde{2}^{-1} & \tilde{4}^{-1} & \tilde{1} \end{bmatrix} \end{array} \Rightarrow w_E = \begin{bmatrix} w_{51} \\ w_{52} \\ w_{53} \\ w_{54} \end{bmatrix} = \begin{bmatrix} (0.134, 0.478, 1.298) \\ (0.072, 0.212, 0.649) \\ (0.101, 0.232, 0.649) \\ (0.038, 0.078, 0.303) \end{bmatrix}$$

By fuzzy multiplication of the related sub-criteria weighting vectors and criteria weights, as given in Eq. 13, the aggregated weights of the sub-criteria were obtained with respect to the goal. For example:

$$w_A^{'} = w_1 \otimes w_A = \begin{bmatrix} w_1 \otimes w_{11} \\ w_1 \otimes w_{12} \\ w_1 \otimes w_{13} \\ w_1 \otimes w_{14} \\ w_1 \otimes w_{15} \\ w_1 \otimes w_{16} \end{bmatrix} = \begin{bmatrix} (0.090, 0.265, 0.728) \otimes (0.073, 0.226, 0.662) \\ (0.090, 0.265, 0.728) \otimes (0.042, 0.108, 0.441) \\ (0.090, 0.265, 0.728) \otimes (0.033, 0.071, 0.343) \\ (0.090, 0.265, 0.728) \otimes (0.067, 0.271, 0.809) \\ (0.090, 0.265, 0.728) \otimes (0.060, 0.219, 0.735) \\ (0.090, 0.265, 0.728) \otimes (0.030, 0.104, 0.294) \end{bmatrix}$$

$$= \begin{bmatrix} (0.007, 0.060, 0.482) \\ (0.004, 0.029, 0.321) \\ (0.003, 0.019, 0.250) \\ (0.006.0.072, 0.589) \\ (0.005, 0.058, 0.535) \\ (0.003, 0.028, 0.214) \end{bmatrix}$$

Table 2 Final ranking of management plans

Decision alternative	Index of optimism			Final rank
	λ=0.0 (pessimistic)	λ=0.5 (moderate)	λ=1.0 (optimistic)	
Plan 1	0.388	0.366	0.365	1
Plan 2	0.280	0.304	0.305	3
Plan 3	0.332	0.330	0.330	2

The set of vectors $w_A^{'}$ through to $w_E^{'}$ consists of the fuzzy weights of all sub-criteria with respect to the goal as defined by Eq. 14. Therefore:

$$W' = \left\{ w_A^{'}, w_B^{'}, w_C^{'}, w_D^{'}, w_E^{'} \right\}$$

is the vector represented by Eq. 15 with the total of 24 entries. Note that here we have k_1= 6, k_2=4, k_3=4, k_4=6, k_5=4 which gives the total of K=24 sub-criteria to be used when evaluating the performance ratings of alternatives.

The assessment of alternatives has been performed by using relations 16–18, followed by the AHP synthesis by using relations 15 and 19. The final alternative performance weights, with respect to the overall goal, have been calculated by Eq. 20 as:

$$F = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix} = \begin{bmatrix} (0.016, 0.388, 9.004) \\ (0.012, 0.280, 7.556) \\ (0.014, 0.332, 8.147) \end{bmatrix}.$$

For the typical values of λ that express the decision maker's attitude toward risk, the final ranking of alternative management plans is obtained by applying Eq. 21. The normalised values presented in Table 2 show that Plan 1 is the best. It is followed by Plan 3 and Plan 2, regardless of the decision maker's level of optimism.

By using the centre of gravity method to defuzzify the F values given above, the final weights of alternatives obtained after normalization were: 0.365 (Plan 1), 0.305 (Plan 2) and 0.330 (Plan 3). Obviously, the final ranking is equal to the previous one.

To recall, according to the management Plan 1 demands related to human supply and animal supply should be fully satisfied in the future at present level needs. The remaining waters should be used in a way to give priority to irrigation by strictly following future needs. In the case of any surplus waters, ecological demands should be satisfied.

For the sake of completeness, the original version of AHP has been used with the same judgment matrices. Crisp values were used instead of fuzzy values, and both the distributive and the ideal mode of the method produced similar results. For example, for the distributive mode the final ranking was the same and the derived weights were: 0.409 (Plan 1), 0.278 (Plan 2) and 0.313 (Plan 3). In turn, the overall inconsistency of the decision process was computed as 0.06, which is below tolerant 0.10. Although it does not prove that the related FDM application is also consistent, the indication obtained in this way is that it may be considered so.

6 Conclusion

AHP has been proven to be an efficient method in tackling a multi-criteria, decision making problem whatever its formulation and solving framework is – crisp or fuzzy. However, related versions of the method all suffer from shortcomings such as unbalanced ratios of estimations, the strong influence of subjective judgments on final results and exposure of the method to inconsistencies. This paper presents an approach which aims to improve the application of the fuzzy version of AHP in real water management situations. By following the logic of the original (crisp) method in solving decision problems with criteria, sub-criteria and alternatives, a well-balanced fuzzy framework has been created. It fuzzifies Saaty's 9-point fundamental scale at the beginning of the computational procedure and assesses the importance of criteria and sub-criteria once they have been mapped into a unique level. The evaluation of an alternative performance with respect to 'unified' criteria and sub-criteria leads to the derivation of a unique fuzzy performance matrix. Fuzzy extent analysis is employed in all assessments. Finally, an integral value method of defuzzifying the results is proposed for the fast and reliable completion of the procedure.

An example of ranking water management scenarios for large semi-arid basin of the Paraguacu River in Brazil has been presented. Five criteria with 24 sub-criteria have been used for assessing three different management plans. The proposed fuzzy decision making (FDM) approach has been verified as computationally efficient and stable. The derived results have been checked by an alternative (centre of gravity) method of defuzzification and the same ranking of management plans has been obtained. Finally, the standard and revised (ideal mode) versions of AHP, which both use eigenvector method to derive weights of the decision elements, are used to check the consistency of the overall decision making process. The consistency was well below the tolerant limit, and again the final ranks of management plans were equal to those derived via the FDM approach.

Due to the satisfactory results of performed tests, the FDM approach can be considered to be flexible and robust. In particular, it has been recommended as a reliable support tool for use by decision makers in real situations, characterised by the uncertainty and imprecision of both the problem and the decision maker's expertise and cognitive abilities. One of the expected advantages of this is the ease of implementing the proposed method in a meeting with stakeholders.

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