Modern Physics Letters A
Vol. 20, No. 13 (2005) 965–975

CRITICAL BEHAVIOR OF THE COMPACTIFIED GINZBURG–LANDAU MODEL

A. P. C. MALBOUISSON*

Centre de Physique Théorique, Ecole Polytechnique, 91128 Palaiseau, France
adolfo@cbpf.br

J. M. C. MALBOUISSON†, A. E. SANTANA‡ and F. C. KHANNA§

Department of Physics, University of Alberta, Edmonton, AB T6G 2J1, Canada
†*jmalboui@ufba.br*
‡*asantana@fis.unb.br*
§*khanna@phys.ualberta.ca*

Received 2 February 2005

In a field theoretical framework, we study the N-component Ginzburg–Landau model compactified in one of the spatial dimensions. Taking the large-N limit, including one-loop corrections to the coupling constant, we calculate the transition temperature (T_c) for a system bounded between two parallel planes as a function of the separation distance (L) between then. We show that $T_c(L)$ decreases as the separation is diminished in a slightly nonlinear way. The minimal separation for the suppression of the second-order transition is lower than the one obtained without considering coupling-constant corrections.

Keywords: Compactified Ginzburg–Landau model; second-order phase transition.

PACS Nos.: 11.10.-z, 74.20.De

1. Introduction

In a recent study, a generalized compactification formalism was applied for the first time to treat the Euclidean massive $(\lambda\varphi^4)_D$ model compactified in a d-dimensional ($d \leq D$) subspace.[1] Such a formalism allowed us to extend previous results in the effective potential framework for finite temperature and spatial boundaries, generalizing and unifying results from works on the behavior of field theories in the presence of spatial planar boundaries[2] and finite temperature.[3]

*Centro Brasileiro de Pesquisas Físicas, 22290-180, Rio de Janeiro, RJ, Brazil
†Instituto de Física Universidade Federal da Bahia, 40210-340, Salvador, BA, Brazil
‡Instituto de Física, Universidade de Brasília, 70919-9700, Brasília, DF, Brazil
§TRIUMF, 4004, Wesbrook Mall, Vancouver, BC V6T 2A3, Canada

In the present work our aim is to investigate, in a field theoretical framework, effects associated to spatial confinement for a model that permits a nonperturbative approach. We consider the N-component Euclidean $\lambda\varphi^4_D$ theory at leading order in $1/N$, the system being submitted to the constraint of confinement between two parallel planes a distance L apart from one another. From a physical point of view, for $D = 3$ and introducing temperature by means of the mass term in the Hamiltonian, this corresponds to a film-like material described by the N-component Ginzburg–Landau model. The large-N limit allows us to incorporate the L-corrections to the coupling constant in a nonperturbative manner.

The study of effects of spatial boundaries on the behavior of physical systems appears in several forms in the literature. For instance, there are systems that present defects, as domain walls. At the level of effective field theories, in many cases this can be modeled by considering a Dirac field whose mass changes sign as it crosses the defect, which means that the domain wall can be interpreted as a kind of a critical boundary.[4,5]

Another very relevant example concerns the Casimir effect. The ideas of compactification and a generalization of the Bogoliubov transformation, the latter to include finite temperature effects specifically, have been successfully applied to the study of the Casimir effect for bosonic[6] and fermionic fields.[7] These mechanisms make computations, for the simple case of two parallel plates separated by a distance L, much simpler than the early attempts.[8] This approach has also been extended to a three-dimensional box maintained at temperature T.[7]

Questions concerning the existence and stability of phase transitions may also be raised if we inquire about the behavior of field theories as a function of spatial boundaries. The existence of phase transitions would be in this case also associated with some spatial parameters describing the breaking of translational invariance, in our case the distance L between planes confining the system. In particular the question of how the critical temperature for a second-order phase transition depends on the thickness of the film can be raised. In this situation, for Euclidean field theories a generalized Matsubara formalism applies for the breaking of invariance along spatial directions.

A central ingredient in our approach is the topological nature of the Matsubara imaginary-time formalism. To calculate the partition function in a quantum field theory, the Matsubara prescription is equivalent to a path-integral approach on $R^{D-1} \times S^1$, where S^1 is a circle of circumference $\beta = 1/T$. This result was demonstrated at the one-loop level by Polchinski[9] and has been assumed to be valid for higher orders.[10] As a consequence, the Matsubara formalism can be considered, in a generalized way, as a mechanism to deal also with spatial constraints in a field theory model. In such a case, for consistency, the fields fulfill periodic (antiperiodic) boundary conditions for bosons (fermions). This procedure has been applied in different physical situations, for bosons[1,2] and for fermions.[11]

In this paper we investigate how the physically relevant quantities, coupling constant, mass, and in particular the critical temperature is affected by the compactification of one of the spatial dimensions. In Sec. 2, we calculate the effective potential for the compactified N-component Ginzburg–Landau model. Next, the renormalization in the large-N limit is implemented in Sec. 3, where formulas for the renormalized L-dependent coupling constant and mass are presented. Finally, in Sec. 4, after a mass renormalization, the critical curve relating the transition temperature and the film thickness is established and comparison is made with the previous results for standard Ginzburg–Landau model without coupling-constant corrections.[12] Some concluding remarks and comments are made in the last section.

2. The Effective Potential for the Compactified Ginzburg–Landau Model

We consider the N-component vector model described by the Ginzburg–Landau Hamiltonian density,

$$\mathcal{H} = \frac{1}{2}(\nabla\varphi)^2 + \frac{1}{2}m_0^2\varphi_a\varphi_a + u(\varphi_a\varphi_a)^2\,, \tag{1}$$

in Euclidean D-dimensional space, where u is the coupling constant, $m_0^2 = \alpha(T-T_0)$ is the bare mass (T_0 being the bulk transition temperature) and summation over repeated indices a is assumed. In the following we will consider the model described by the Hamiltonian (1) with N components and take the large N limit with $Nu = \lambda$ *fixed*. We consider the system confined between two parallel planes, normal to the x-axis, a distance L apart from one another and use Cartesian coordinates $\mathbf{r} = (x, \mathbf{z})$, where $\mathbf{z}$ is a $(D-1)$-dimensional vector, with corresponding momenta $\mathbf{k} = (k_x, \mathbf{q})$, $\mathbf{q}$ being a $(D-1)$-dimensional vector in momentum space. In this case, the model is supposed to describe a film of thickness L.

Under these conditions the generating functional of the correlation functions is written as,

$$\mathcal{Z} = \int \mathcal{D}\varphi \exp\left[-\int_0^L dx \int d^{D-1}\mathbf{z}\,\mathcal{H}(\varphi, \nabla\varphi)\right]\,, \tag{2}$$

with the field $\varphi(x, \mathbf{z})$ satisfying the condition of confinement along the x-axis, $\varphi(x = 0, \mathbf{z}) = \varphi(x = L, \mathbf{z}) = 0$. Then the field should have a mixed series-integral Fourier expansion of the form,

$$\varphi(x, \mathbf{z}) = \sum_{n=-\infty}^{\infty} \int d^{D-1}\mathbf{q}\, e^{-i\omega_n x - i\mathbf{q}\cdot\mathbf{z}}\tilde{\varphi}(\omega_n, \mathbf{q})\,, \tag{3}$$

where $\omega_n = 2\pi n/L$. The above conditions of confinement allow us to proceed with respect to the x-coordinate, in a manner analogous to the imaginary-time Matsubara formalism in field theory.[9,10] The Feynman rules are modified following the prescription,

$$\int \frac{dk_x}{2\pi} \to \frac{1}{L}\sum_{n=-\infty}^{+\infty}\,, \qquad k_x \to \frac{2n\pi}{L} \equiv \omega_n\,. \tag{4}$$

We emphasize that here we are considering a Euclidean field theory in D *purely* spatial dimensions, we are *not* working in the framework of finite temperature field theory. Temperature is introduced in the mass term of the Hamiltonian by means of the usual Ginzburg–Landau construction. The Matsubara mechanism is used, exploring its topological interpretation, to engender the spatial compactification.

In the following, to deal with dimensionless quantities in the regularization procedures, we introduce parameters $c^2 = m^2/4\pi^2\mu^2$, $b = (L\mu)^{-2}$, $g = u/8\pi^2\mu^{4-D}$ and $\phi_0^2 = \varphi_0^2/\mu^{D-2}$, where m is the *renormalized* mass in the absence of boundaries, φ_0 is the normalized vacuum expectation value of the field (the classical field) and μ is a mass scale (naturally, the results do not depend on μ). In terms of these parameters, we start from the expression for the one-loop contribution to the effective potential in the absence of constraints,

$$U_1(\phi, L = \infty) = \mu^D \sum_{s=1}^{\infty} \frac{(-1)^{s+1}}{2s} g^s \phi_0^{2s} \int \frac{d^D k'}{(k'^2 + c^2)^s}\,, \tag{5}$$

where $k' = k/2\pi\mu$ is dimensionless. Then performing the replacements (4), the compactified (L-dependent) one-loop contribution to the effective potential can be written as

$$U_1(\phi, L) = \mu^D \sqrt{b} \sum_{s=1}^{\infty} \frac{(-1)^s}{2s} g^s \phi_0^{2s} \sum_{n=-\infty}^{+\infty} \int \frac{d^{D-1} q'}{(bn^2 + c^2 + \mathbf{q}'^2)^s} \tag{6}$$

or, using a well-known dimensional regularization formula[13]

$$\int \frac{d^d p}{(2\pi)^d} \frac{1}{(p^2 + M)^s} = \frac{\Gamma\left(s - \frac{d}{2}\right)}{(4\pi)^{d/2}\Gamma(s)} \frac{1}{M^{s-d/2}}\,, \tag{7}$$

in the form

$$U_1(\phi, L) = \mu^D \sqrt{b} \sum_{s=1}^{\infty} f(D, s) g^s \phi_0^{2s} Z_1^{c^2}\left(s - \frac{D-1}{2}; b\right), \tag{8}$$

where $f(D, s)$ is a function proportional to $\Gamma(s - \frac{D-1}{2})$ and $Z_1^{c^2}(s - \frac{D-1}{2}; b)$ is one of the Epstein–Hurwitz *zeta*-functions[14] defined by

$$Z_K^{c^2}(\nu; b_1, \ldots, b_K) = \sum_{\{n_j\}=-\infty}^{+\infty} (b_1 n_1^2 + \cdots + b_K n_K^2 + c^2)^{-\nu}\,, \tag{9}$$

which is valid for $\mathrm{Re}(\nu) > K/2$ (in our case $\mathrm{Re}(s) > D/2$). The Epstein–Hurwitz *zeta*-function can be extended to the whole complex s-plane and we obtain, after some manipulations,[2] the one-loop correction to the effective potential,

$$U_1(D, L) = \sum_{s=1}^{\infty} u^s \varphi_0^{2s} h(D, s) \times \left[2^{-(\frac{D}{2}-s+2)} \Gamma\left(s - \frac{D}{2}\right) m^{D-2s} + \sum_{n=1}^{\infty} \left(\frac{m}{nL}\right)^{\frac{D}{2}-s} K_{\frac{D}{2}-s}(mnL)\right], \tag{10}$$

where

$$h(D,s) = \frac{1}{2^{D/2+2s-1}\pi^{D/2}} \frac{(-1)^{s+1}}{s\Gamma(s)} \tag{11}$$

and $K_{\frac{D}{2}-s}$ are Bessel functions of the third kind.

The above one-loop results, in the effective potential framework, have been obtained by the concurrent use of dimensional and *zeta*-function analytic regularizations in evaluating formally the integral over the continuous momenta and the summation over discrete frequencies. We get sums of polar (L-independent) terms plus L-dependent analytic corrections. Renormalized quantities are obtained by subtraction of the divergent (polar) terms appearing in the quantities obtained by application of the modified Feynman rules and dimensional regularization formulas. These polar terms are proportional to Γ-functions having the dimension D in the argument and correspond to the introduction of counterterms in the original Lagrangian density. In order to have a coherent renormalization scheme in any dimension, these subtractions should be performed even in the case of those values of the dimension D where no poles of Γ-functions are present. In these cases a finite renormalization is performed.

3. Renormalization in the Large-N Limit

In the following, we consider the four-point function at zero external momenta, which we take as the basic object for our definition of the renormalized coupling constant. At leading order in $1/N$, it is given by

$$\Gamma_D^{(4)}(\mathbf{p}=0,m,L) = \frac{u}{1+Nu\Pi(D,m,L)}\,, \tag{12}$$

where $\Pi(D,m,L) = \Pi(\mathbf{p}=0,D,m,L)$ corresponds to the single bubble diagram,

$$\Pi(D,m,L) = \frac{1}{L}\sum_{n=-\infty}^{\infty}\int \frac{d^{D-1}q}{(2\pi)^{D-1}} \frac{1}{[\mathbf{q}^2+\omega_n^2+m^2]^2}\,. \tag{13}$$

To proceed we use the renormalization conditions

$$\left.\frac{\partial^2}{\partial\varphi_0^2}U(D,L)\right|_{\varphi_0=0} = m^2 \tag{14}$$

and

$$\left.\frac{\partial^4}{\partial\varphi_0^4}U(D,L)\right|_{\varphi_0=0} = u\,, \tag{15}$$

from which we deduce formally that the single bubble function $\Pi(D,m,L)$ is obtained from the coefficient of the fourth power of the field ($s=2$) in Eq. (10), affected by the minus sign. In general, such a coefficient is ultraviolet divergent and a renormalization procedure is needed. Then, using Eq. (15), we can write

$\Pi(D,m,L)$ in the form

$$\Pi(D,m,L) = H(D,m) + G(D,m,L)\,, \tag{16}$$

where the L-dependent term $G(D,m,L)$ comes from the second term between brackets in Eq. (10),

$$G(D,m,L) = \frac{3}{2(2\pi)^{D/2}} \sum_{n=1}^{\infty} \left[\frac{m}{nL}\right]^{(D-4)/2} K_{\frac{D-4}{2}}(nLm)\,, \tag{17}$$

and $H(D,m)$ is a polar term coming from the first term between brackets in Eq. (10),

$$H(D,m) \propto \Gamma\left(2 - \frac{D}{2}\right) m^{D-4}\,. \tag{18}$$

We see from Eq. (18) that for even dimensions $D \geq 4$, $H(D,m)$ is divergent, due to the pole of the Γ-function. Accordingly this term must be subtracted to give the renormalized single bubble function $\Pi_R(D,m,L)$. We get simply,

$$\Pi_R(D,m,L) = G(D,m,L)\,. \tag{19}$$

As mentioned above, in order to have a coherent procedure for a generic dimension D, the subtraction of the term $H(D,m)$ should be performed even in the case of odd dimensions, where no poles of Γ-functions are present. From the properties of Bessel functions, it can be seen from Eq. (17) that for any dimension D, $G(D,m,L)$ satisfies the conditions

$$\lim_{L\to\infty} G(D,m,L) = 0\,, \qquad \lim_{L\to 0} G(D,m,L) \to \infty\,. \tag{20}$$

We also conclude, from the properties of Bessel functions, that $G(D,m,L)$ is positive for all values of D and L.

Let us define the L-dependent renormalized coupling constant $u_R(m,D,L)$, at the leading order in $1/N$, as

$$\Gamma^{(4)}_{D,R}(\mathbf{p}=0,m,L) \equiv u_R(D,m,L) = \frac{u}{1 + Nu\Pi_R(D,m,L)} \tag{21}$$

and the renormalized coupling constant in the absence of constraints as

$$u_R(D,m) = \lim_{L\to\infty} \Gamma^{(4)}_{D,R}(\mathbf{p}=0,m,L)\,. \tag{22}$$

From Eqs. (22), (21) and (20) we get simply $u_R(D,m) = u$. In other words, we have done a choice of renormalization scheme such that the constant u introduced in the Hamiltonian corresponds to the *renormalized* coupling constant. From Eqs. (21) and (19) we obtain the L-dependent renormalized coupling constant

$$Nu_R(D,m,L) \equiv \lambda_R(D,m,L) = \frac{\lambda}{1 + \lambda G(D,m,L)}\,, \tag{23}$$

where we have defined the *fixed* coupling constant $\lambda = Nu$ (see comments after Eq. (1)).

It is to be noted that we have used a modified minimal subtraction scheme where the mass and coupling constant counterterms are poles at the physical values of s. The L-dependent correction to the coupling constant is proportional to the regular part of the analytical extension of the inhomogeneous Epstein *zeta*-function in the neighborhood of the pole at $s = 2$. The same argument applies to the mass renormalization, the L-dependent physical mass at one-loop approximation being given by

$$m^2(D,L) = m_0^2 + \frac{\lambda}{2(2\pi)^{\frac{D}{2}}} \sum_{n=1}^{\infty} \left[\frac{m_0}{nL}\right]^{\frac{D-2}{2}} K_{\frac{D-2}{2}}(nLm_0)\,; \tag{24}$$

for $D = 3$, using that $K_{\frac{1}{2}}(z) = \sqrt{\frac{\pi}{2z}}e^{-z}$, we get the closed formula

$$m^2(D=3,L) = m_0^2 - \frac{\lambda}{\pi L}\log(1 - e^{-m_0 L})\,. \tag{25}$$

4. Mass Behavior and Critical Curve

We shall now obtain the critical curve and determine the dependence of the critical temperature, $T_{\rm c}$, with the thickness of a film. If we start in the ordered phase with a negative squared mass, the model exhibits spontaneous symmetry breaking of the $O(N)$ symmetry to $O(N-1)$, but for sufficiently small critical values of T^{-1} and L the symmetry is restored. We can define the critical curve $C(T_c, L) = 0$ as the curve in the $T \times L$ plane for which the inverse squared correlation length, $\xi^{-2}(T, L, \phi_0)$, vanishes in the large-N gap equation

$$\begin{aligned}\xi^{-2}(T,L,\phi_0) &= m_0^2 + 2\lambda_R(D,T,L)\phi_0^2 \\ &\quad + \frac{2\lambda_R(D,T,L)}{L}\sum_n \int \frac{d^{D-1}q}{(2\pi)^{D-1}}\,\frac{1}{\mathbf{q}^2 + \omega_n^2 + \xi^{-2}(T,L,\phi_0)}\,,\end{aligned} \tag{26}$$

where ϕ_0 is the normalized vacuum expectation value of the field (different from zero in the ordered phase). In the disordered phase, in particular in the neighborhood of the critical curve, ϕ_0 vanishes and the gap equation reduces to a L-dependent Dyson–Schwinger equation which, after performing steps analogous to those leading from Eq. (6) to Eq. (10), can be written in the form

$$m^2(D,T,L) = m_0^2 + \frac{\lambda_R(D,T,L)}{2(2\pi)^{D/2}} \sum_{n=1}^{\infty} \left[\frac{m(D,T,L)}{nL}\right]^{\frac{D-2}{2}} K_{\frac{D-2}{2}}(nLm(D,T,L))\,. \tag{27}$$

In Eqs. (26) and (27) $\lambda_R(D, L)$ is the renormalized L-dependent coupling constant, which is itself a function of $m(D, T, L)$ given by appropriate versions of Eqs. (23) and (17), i.e.

$$\lambda_R(D,T,L) = \frac{\lambda}{1 + \lambda G(D,T,L)}\,, \tag{28}$$

with

$$G(D,T,L) = \frac{3}{2(2\pi)^{D/2}} \sum_{n=1}^{\infty} \left[\frac{m(D,T,L)}{nL}\right]^{\frac{D-4}{2}} K_{\frac{D-4}{2}}(nLm(D,T,L)) \,. \tag{29}$$

Therefore $m(D,T,L)$ is given by a complicated set of equations, Eqs. (27)–(29), since $\lambda_R(D,T,L)$ depends on $m(D,T,L)$. Nevertheless, limiting ourselves to the neighborhood of criticality, $m^2(D,T,L) \approx 0$, we may investigate the behavior of the system by using in Eq. (27), Eqs. (28) and (29) the asymptotic formula for small values of the argument of the Bessel function,

$$K_\nu(z) \approx \frac{1}{2}\Gamma(\nu)\left(\frac{z}{2}\right)^{-\nu}, \qquad z \sim 0\,; \quad \mathrm{Re}(\nu) > 0\,, \tag{30}$$

which leads to

$$m^2(D,T,L) \approx m_0^2 + \frac{\lambda_R(D,T,L)}{(2\pi)^{D/2}}\Gamma\left(\frac{D}{2}-1\right)L^{2-D}\zeta(D-2)\,, \tag{31}$$

where $\zeta(D-2) = \sum_{n=1}^{\infty}(1/n^{D-2})$ is the Riemann *zeta*-function, defined for $D > 3$. Similarly, inserting Eq. (30) into Eqs. (28) and (29), $\lambda_R(D,T,L)$ can be written for $m^2(D,T,L) \approx 0$ as

$$\lambda_R(D,L) \approx \frac{\lambda}{1+\lambda C(D)L^{4-D}\zeta(D-4)}\,, \tag{32}$$

where $C(D) = \frac{3}{4\pi^{D/2}}\Gamma(\frac{D-4}{2})$. Taking $m(D,T,L) = 0$ and $m_0^2 = \alpha(T-T_0)$ in Eq. (31), we obtain, in the large-N limit, the critical curve in the $T \times L$ plane for Euclidean space dimension D ($D > 3$),

$$\alpha(T_{\mathrm{c}} - T_0) + \frac{\lambda_R(D,L)}{(2\pi)^{D/2}}\Gamma\left(\frac{D}{2}-1\right)L^{2-D}\zeta(D-2) = 0\,. \tag{33}$$

For $D = 3$ the Riemann *zeta*-function has a pole. We cannot obtain a critical curve in dimension $D \leq 3$ by a limiting procedure from Eq. (33). For $D = 3$, which corresponds to the physically interesting situation of the system confined between two parallel planes embedded in a three-dimensional Euclidean space, Eq. (33) becomes meaningless. To obtain a critical curve in $D < 3$, we perform an analytic continuation of the *zeta*-function $\zeta(z)$ to values of the argument $z < 1$, by means of the reflection property[15]

$$\zeta(z) = \frac{1}{\Gamma(z/2)}\Gamma\left(\frac{1-z}{2}\right)\pi^{z-\frac{1}{2}}\zeta(1-z)\,, \tag{34}$$

which defines a meromorphic function having only one simple pole at $z = 1$. For $D = 3$, to get a physically meaningful result, a subtraction procedure is needed and

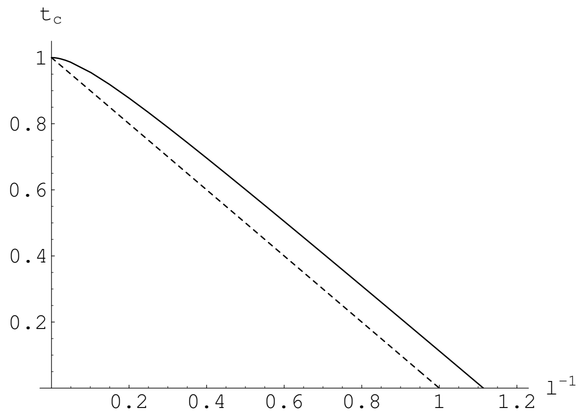

Fig. 1. Reduced transition temperature, $t_c = T_c/T_0$, for films as a function of the inverse of the reduced thickness, $l^{-1} = L_0/L$ (with $L_0 = \gamma\lambda/2\sqrt{2}\pi\alpha T_0$), fixing $\lambda L_0 = 10$. The dashed line corresponds to $t_c(l) = 1 - l^{-1}$, obtained without considering L-corrections to the coupling constant in the Ginzburg–Landau model.

can be done as follows: remembering the formula,

$$\lim_{z\to 1}\left[\zeta(z) - \frac{1}{z-1}\right] = \gamma\,, \tag{35}$$

where $\gamma \cong 0.57$ is the Euler–Mascheroni constant, we define from Eq. (31) the *renormalized mass* $\bar{m}$ as,

$$\begin{aligned}\bar{m}^2(T,L) &= \lim_{D\to 3_-}\left[m^2(D,T,L) - \frac{\lambda_R(D=3,L)}{2\pi\sqrt{2}L^{D-2}(3-D)}\right]\\ &= \alpha(T-T_0) + \frac{\lambda_R(D=3,L)\gamma}{2\pi\sqrt{2}L}\,.\end{aligned} \tag{36}$$

Taking this *renormalized mass* equal to zero leads to the critical curve in dimension $D = 3$ which, using Eqs. (32) and (34) to evaluate $\lambda_R(D = 3, L)$, can be written as

$$T_c(L) = T_0 - \frac{2\sqrt{2}\gamma\lambda}{8\pi\alpha L + \alpha\lambda L^2}\,. \tag{37}$$

It is interesting to compare this result with the critical curve for a film deduced from the Ginzburg–Landau model in which the L-correction to the coupling constant is neglected, obtained from Eq. (36) by taking $\lambda_R = \lambda$. In this lowest level of approximation, the critical temperature is simply a linear decreasing function of $1/L$, which is plotted in Fig. 1, together with the critical curve (37), for comparison.

We find that the critical temperature (37) decreases from T_0 (the bulk transition temperature) as L diminishes reaching zero for a minimal thickness $L^{(0)}$, below which the transition is suppressed. This minimal thickness is given by

$$L^{(0)} = \frac{4\pi}{\lambda}\left[\sqrt{1 + \frac{\lambda L_0}{2\pi}} - 1\right]\,, \tag{38}$$

where $L_0 = \gamma\lambda/(2\sqrt{2}\pi\alpha T_0)$ is the minimal thickness for the existence of the ordered phase in the bare approximation of neglecting L-corrections to the coupling constant. This minimal thickness (not considering coupling constant corrections) coincides with the result for the standard (two-component) Ginzburg–Landau model,[12] except for a simple symmetry factor. We also find that the predicted minimal film thickness, for the N-component model including the L-correction to the coupling constant, is lower than the value L_0 it would have without finite thickness corrections to the coupling constant, but the general behavior of the curves $t_c = t_c(l)$, in both cases, is very similar.

Notice that the results obtained here might be applicable to any physical system undergoing a second-order phase transition described by the Ginzburg–Landau model. For example, the decrease of the transition temperature with the inverse of the film thickness (as described above) has been experimentally observed for superconductors.[16,17] In fact, our results do not depend on particular physical systems, appearing only as a topological consequence of the compactification in one spatial dimension of the Ginzburg–Landau model.

5. Concluding Remarks

In this work we have discussed the N-component Ginzburg–Landau model, in the large-N limit, the system being confined between two parallel planes. This model is assumed to describe a film undergoing a second-order phase transition. We find that the transition temperature is a decreasing function of the inverse of the film thickness and that there is a minimal thickness, below which the transition disappears. This minimal thickness was determined in terms of the Ginzburg–Landau parameters and the bulk transition temperature. We verified that the inclusion of boundary corrections to the coupling constant leads to a lower least-thickness for the film to sustain the existence of the ordered phase.

It is worthwhile to notice that such a decreasing behavior of the critical temperature with the inverse of the film thickness has been observed experimentally. We would like to emphasize that our results are completely independent of the microscopic characteristics of the physical system considered. They emerge solely as a topological effect of the compactification of one of the spatial dimensions in the Ginzburg–Landau model and are obtained in a purely field theoretical framework.

Acknowledgments

This paper was partially supported by CAPES and CNPq (Brazil), and by NSERC (Canada). One of us (APCM) thanks FAPERJ (Brazil) for partial support. Two of us (JMCM and AES) thank the Theoretical Physics Institute and Vice-President (Research) for partial support during their stay at the University of Alberta.

References

1. A. P. C. Malbouisson, J. M. C. Malbouisson and A. E. Santana, *Nucl. Phys.* **B631**, 83 (2002).
2. A. P. C. Malbouisson and J. M. C. Malbouisson, *J. Phys. A Math. Gen.* **35**, 2263 (2002).
3. G. N. J. Añaños, A. P. C. Malbouisson and N. F. Svaiter, *Nucl. Phys.* **B547**, 221 (1999).
4. L. Da Rold, C. D. Fosco and A. P. C. Malbouisson, *Nucl. Phys.* **B624**, 485 (2002).
5. C. D. Fosco and A. Lopez, *Nucl. Phys.* **B538**, 685 (1999).
6. J. C. da Silva, F. C. Khanna, A. Matos Neto and A. E. Santana, *Phys. Rev.* **A66**, 052101 (2002).
7. H. Queiroz, J. C. da Silva, F. C. Khanna, J. M. C. Malbouisson, M. Revzen and A. E. Santana, *Ann. Phys.* **317**, 220 (2005).
8. L. S. Brown and G. J. Maclay, *Phys. Rev.* **184**, 1272 (1969).
9. J. Polchinski, *Commun. Math. Phys.* **104**, 37 (1986).
10. J. J. Atick and E. Witten, *Nucl. Phys.* **B310**, 291 (1988).
11. A. P. C. Malbouisson, J. M. C. Malbouisson, A. E. Santana and J. C. Silva, *Phys. Lett.* **B583**, 373 (2004).
12. A. P. C. Malbouisson, J. M. C. Malbouisson and A. E. Santana, *Phys. Lett.* **A318**, 406 (2003).
13. J. Zinn-Justin, *Quantum Field Theory and Critical Phenomena* (Clarendon Press, 1996).
14. A. Elizalde and E. Romeo, *J. Math. Phys.* **30**, 1133 (1989); see also E. Elizalde, S. D. Odintsov, A. Romeo, A. A. Bytsenko and S. Zerbini, *Zeta Regularization Techniques with Applications* (World Scientific, 1994).
15. Eds. M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions* (Dover, 1965).
16. D. G. Naugle and R. E. Glover, *Phys. Lett.* **A28**, 611 (1969).
17. M. S. M. Minhaj, S. Meepagala, J. T. Chen and L. E. Wenger, *Phys. Rev.* **B49**, 15235 (1994); H. Raffy, R. B. Laibowitz, P. Chaudhari and S. Maekawa, *ibid.* **B28**, R6607 (1983).