TUNNELING OF SEISMIC BODY WAVES THROUGH THIN HIGH-VELOCITY LAYERS IN COMPLEX STRUCTURES

Vlastislav Červený
Institute of Geophysics, Charles University, Prague*)

Paulo R. A. Aranha
Instituto de Geociências, Universidade Federal da Bahia, Salvador**)

Summary: The hybrid ray-reflectivity method is applied to the problem of the transmission of the reflected wave field through a thin high-velocity layer (or through a thin stack of high velocity layers), situated in the overburden of the reflector. In the hybrid ray-reflectivity method, the standard ray method is applied in the smooth parts of the model, and the reflectivity method is used locally at the thin high-velocity layer. With the exception of small epicentral distances, the standard ray method itself fails in such computations. The reason is that a considerable part of the energy for overcritical angles of incidence may be tunneled through the thin high-velocity layer along complex ray-paths, corresponding to inhomogeneous waves. The reflectivity method, applied locally at the thin high-velocity layer, automatically includes all inhomogeneous wave contributions. Thus, the hybrid ray-reflectivity method removes fully the limitations of the standard ray method, but still retains its main advantages, such as its applicability to 2-D and 3-D complex layered structures, flexibility, and low-cost computations. In the numerical examples, the hybrid ray-reflectivity synthetic seismograms are compared with standard ray synthetic seismograms and with full reflectivity computations. The numerical examples show that the hybrid ray-reflectivity method describes the tunneling of seismic energy through a thin high-velocity layer with sufficient accuracy.

1. INTRODUCTION

The hybrid ray-reflectivity method can be used to compute body wave synthetic seismograms in 2-D and 3-D laterally varying layered structures containing thin transition layers, see [10]. By a thin transition layer we understand a layer the thickness of which is roughly 1/2 or less of the prevailing wavelength. In the hybrid ray-reflectivity method, ray calculations are applied to those parts of the model which do not contain the thin high-velocity layers and which are assumed to be smooth. On the contrary, matrix (reflectivity) computations are applied locally to the thin transition layers. The BEAM87 program package, designed for such hybrid computations in 2-D laterally varying layered structures containing a thin transition layer, was described in [10]. The method and the program package were used there to study the P waves reflected from a thin transition layer separating two halfspaces.

In this paper, we apply the hybrid ray-reflectivity method also to reflected P waves, but in a more general configuration. We will not compute the waves reflected from a thin transition layer, but passing through a thin high-velocity layer. The waves are reflected from a single structural first-order interface, and the thin high-velocity layer is situated in the overburden. The thin high-velocity layer is not necessarily homogeneous; it may be represented by a stack of very thin homogeneous layers. It is well known that the standard ray method is not able to treat such a problem, with the exception of small epicentral distances. An important role in

*) Address: Ke Karlovu 3, 121 16 Praha 2.
**) Address: Campus Universitário de Federação, 40210 Salvador, Bahia, Brasil.
the transmission of the seismic energy through the thin high-velocity layer is played by inhomogeneous waves. The inhomogeneous waves do not propagate through the thin layer along real-valued rays, but along complex-valued ray-paths. As the contribution of inhomogeneous waves to the transmission of energy through a thin high-velocity layer may be rather high, the standard ray method itself fails in such computations [23, 9]. Fuchs and Schulz [23] speak of tunneling of seismic energy through the thin layer. These effects are frequency-dependent, the tunneling is stronger for lower frequencies. The hybrid ray-reflectivity method automatically includes all inhomogeneous waves propagating through the thin layer, since the reflectivity algorithm includes them [30]. Thus, the hybrid ray-reflectivity method removes fully this serious limitation of the standard ray method.

In the algorithm of the hybrid ray-reflectivity method, only the ray method in its zero-order approximation is used. The zero-order approximation of the ray method is fully based on the concept of real rays. It is used in many of the computer algorithms and program packages. To simplify the terminology, we speak of the standard ray method. It would, of course, be possible to generalize it by using certain higher-order waves (head waves, etc.). Moreover, it would also be possible to consider complex rays (in addition to real rays). From a theoretical point of view, there is no fundamental problem in the generalization for complex rays [2, 5, 12, 18, 26, 27, 34]. Complex rays have even been used successfully in applications, see, e.g. [25, 33]. The above list is far from complete; many other important references can be found in the papers given above.

However, we have no intention of proposing an algorithm and developing a computer package for the numerical modelling of seismic wave fields using complex rays here. Just the opposite. We merely wish to show that the hybrid ray-reflectivity method can be used to solve certain problem, which involve complex rays, without difficulties.

Various alternatives of the hybrid method used in this paper have been described in the seismological literature. They are designed mostly for one-dimensional computations only [16, 32]. The hybrid ray-reflectivity code proposed by Bernasconi and Drufuca [6] can even be used for a non-horizontal stack of layers. Some of the hybrid methods use the WKBJ method instead of ray computations [1, 15]. Also the method of summation of Gaussian beams can be used instead of the standard ray method [10]. Another hybrid method was proposed for models with localized heterogeneities. It uses finite differences in a heterogeneous region and the frequency-wavenumber method outside the region [19]. The ray method can also be efficiently combined with modal summation. However, we do not intend to review here all the used or possible hybrid methods in the numerical modelling of seismic wave fields; we only wish to say that the hybrid codes are very promising.

To avoid a terminological misunderstanding, it would be useful to add one remark. In the hybrid ray-reflectivity method, by the reflectivity method we understand just matrix computations of frequency-dependent plane-wave reflection/transmission coefficients of a stack of thin layers, for a given angle of incidence. We do not perform any integration over angles of incidence as in the full reflectivity method [22, 30]. Thus, it would perhaps be more suitable to speak about the hybrid ray-matrix method, not about the hybrid ray-reflectivity method. We will, however, follow the terminology introduced in the seismological literature [16]. Only in the Gaussian beam summation method and, consequently, in the hybrid Gaussian beam-reflectivity method, some sort of integration (summation) over angles of incidence is performed.

In Section 2, we will briefly discuss the problem of the transmission of reflected waves through a thin high-velocity layer. In Section 3, we will present several numerical examples and discuss the results from a seismological point of view. In all cases, we will compare the ray synthetic seismograms with hybrid ray-reflectivity synthetic seismograms. To assess the accuracy of the hybrid ray-reflectivity seismograms, we will compare them with the full reflectivity method computations for one simple 1-D model.
2. TUNNELING EFFECTS IN MEDIA CONTAINING THIN
HIGH-VELOCITY LAYERS

Before we present and discuss the results of numerical computation, we will briefly
describe several important properties of homogeneous and inhomogeneous,
reflected and transmitted waves propagating in layered structures. This will help
us to understand better the results of numerical computations. We will mainly con-
centrate our attention on the role of inhomogeneous waves in the transmission of
energy of seismic waves through a thin high-velocity layer. See also [3, 4, 7, 13, 14,
20, 21, 34].

Homogeneous and inhomogeneous reflected and transmitted
waves at a single interface

We consider a plane interface between two homogeneous halfspaces, and a point
source of seismic waves situated in one of these halfspaces. We call the halfspace
containing the source the first (or the upper) halfspace, and the halfspace without
the source the second (or the bottom) halfspace. We denote the compressional and
shear velocities in the first halfspace by $v_1$, $v_i$, and in the second halfspace by $v_2$, $v_2$.
We assume that $v_2 > v_1$.

Let us consider compressional incident, reflected and transmitted waves. We in-
troduce the critical angle $i_1^*$ in a standard way,

$$\sin i_1^* = v_1/v_2.$$  \hspace{1cm} (1)

For a subcritical angle of incidence ($i_1 < i_1^*$), the angle of transmission is real-
valued and a standard transmitted ray-theory wave, with a real-valued ray, is
obtained. For an overcritical angle of incidence, however, the angle of transmission
is complex-valued. The corresponding transmitted wave cannot be studied by the
standard ray method. We call such a wave a transmitted inhomogeneous wave.

The properties of inhomogeneous waves are well known from the seismological
literature, see [7, 31] for the acoustic case and [8] for the elastic case. The most
important property of time-harmonic inhomogeneous waves is that their amplitudes
decrease exponentially with increasing distance from the interface. The exponential
decrease of amplitudes is frequency dependent: for higher frequencies it is faster
and for lower frequencies slower.

In contrast to the transmitted waves, the rays of reflected compressional waves
are always real-valued, for any real-valued angle of incidence of a compressional
incident wave. However, the critical angle still plays an important role even for
reflected waves. We will explain this role from the point of view of the standard ray
approximation. In the subcritical region ($i_1 < i_1^*$), the amplitudes of reflected waves
are usually small. The amplitudes increase rapidly as the angle of incidence $i_1$
approaches the critical angle, $i_1 \to i_1^*$. At the critical angle of incidence, the amplitudes
of the reflected waves reach their maximum. Beyond the critical angle, the amplitudes
are large, and decrease slowly with increasing epicentral distance. The reflection coefficients are real-valued for subcritical angles of incidence, but complex-valued for overcritical angles of incidence. This implies that the shape of the wavelet of the reflected wave is the same as the shape of the wavelet of the incident wave for subcritical angles of incidence. For overcritical angles of incidence, however, the shape of the wavelet of the reflected wave is different from the shape of the wavelet of the incident wave. All these conclusions are only approximate, and valid only in the approximation of the standard ray method. Exact computations yield some well-known differences, namely in the region close to the critical point. In exact computations, the maximum of the amplitude-distance curve of the compressional reflected wave is shifted from the critical point to some distance beyond the critical point. The shift is frequency-dependent, it is small for high frequencies. In addition to the reflected waves, classical head waves are also obtained in the overcritical region. (The head wave is not obtained by the standard ray method, but can also be evaluated by the higher approximation of the ray method.)

Let us add one note to inhomogeneous waves. Consider an inhomogeneous transmitted wave generated by a regular incident wave (with a real-valued ray element between the source and the interface). If we exchange the source and the receiver and use the principle of reciprocity, we obtain waves which propagate as inhomogeneous from the source to the interface and then along a regular ray from the interface to the receiver. Such waves were also described and discussed in the references given above. They were even experimentally verified in seismic laboratory modelling, using a schlieren technique. See [11], where these waves are called pseudospherical waves. At present, such waves are also called “star” waves (P*, S*), see [17]. The inhomogeneous waves propagating from the P-wave point source may also generate regular S waves. Recently, certain S* waves attracted the attention of seismologists, namely the S* wave generated at the Earth’s surface by a P-wave source situated close to the Earth’s surface [4, 5, 21, 24].

Transmission through a thin high-velocity layer

We will first discuss the compressional wave transmitted through the thin layer, without any multiple reflections within the layer. Assume that the P velocities in the upper halfspace and in the bottom halfspace are the same, \( \alpha_1 \). Also assume that the velocity \( \alpha_2 \) in the thin layer separating these two halfspace is higher, \( \alpha_2 > \alpha_1 \). We consider two cases of incidence: the subcritical angle of incidence \( (i_1 < i_1^* ) \) and the overcritical angle of incidence \( (i_1 > i_1^* ) \). The critical angle of incidence \( i_1^* \) is given by (1). The situation is simple for subcritical angles of incidence, see Fig. 1a. The wave propagates through the thin layer along a real-valued ray; the angle between the ray and normal to the interfaces inside the thin layer is given by Snell’s law,

\[
\frac{\sin i_2}{\alpha_2} = \frac{\sin i_1}{\alpha_1}.
\]
For the overcritical angle of incidence, the transmission has a more complicated character. Only an inhomogeneous wave propagates within the thin layer, see Fig. 1b. The inhomogeneous wave, however, changes again into a regular transmitted wave in the bottom halfspace. This is obvious, as Snell's law must be valid at both the upper and lower interfaces of the thin layer; no matter whether the angles are complex or real-valued. Thus, the angle of transmission below the thin layer is again $i_1$. Even though the transmitted wave below the thin layer again propagates along a real-valued ray, it has different properties than the transmitted wave corresponding to the subcritical angle of incidence. It does not propagate along a real-valued ray through the thin layer. We also speak of the tunneling effect and tunnel wave, see Fuchs and Schulz [23]. This type of wave, of course, cannot be calculated by the standard ray method. The amplitude of the transmitted time-harmonic wave for an overcritical angle of incidence (tunnel wave) depends considerably on $h/\lambda$, where $h$ is the thickness of the thin layer, and $\lambda$ is the prevailing wavelength within the layer. The larger $h/\lambda$, the smaller the amplitude; the amplitude decreases exponentially with increasing $h/\lambda$. For small $h/\lambda$ (say, $h/\lambda < 0.5$), the amplitude of the transmitted wave may be rather distinct.

As the amplitudes of the transmitted waves depend on $h/\lambda$, i.e. on frequency, the thin transition layer acts as a low-pass filter at overcritical distances: the lower frequencies propagate easily through the thin layer, but the high frequencies will be mostly filtered out.

Let us emphasize one important point. The above discussion indicates that the angle of transmission $i_1$ in the bottom halfspace cannot exceed the critical angle $i_1^*$ if the standard ray method is used for computation. A real-valued angle of transmission greater than $i_1$ in the lower halfspace can be obtained only if some more sophisticated method is used for computation, in which the inhomogeneous wave propagating within the thin layer is taken into account. As the standard ray method does not consider inhomogeneous waves, angle $i_1$ in the lower halfspace never exceeds $i_1^*$ in the ray computations. Thus, the angles $i_1 > i_1^*$ are screened by the thin high-velocity layer in ray computation. We can also speak of angular filtering by a thin high velocity layer in the standard ray method.
If also shear waves are considered, several different inhomogeneous waves can carry energy through the thin layer. The discussion of such cases, however, is straightforward. The situation is considerably worse if an inner layering inside the thin layer is considered (laminas, etc.). Many multiple reflections, including converted waves, have to be taken into account. It would be impossible, or at least numerically inefficient, to consider all these elementary rays in ray computations. Thus, we have at least two reasons for combining locally the standard ray method with the reflectively method. The reflectivity method automatically yields all multiple reflections and conversions within the thin layer. In addition, it also yields all inhomogeneous waves, which cannot be evaluated by the standard ray method at all.

**Effects of a thin high-velocity layer in the overburden on amplitudes of waves reflected from a first-order interface**

The model under consideration is shown in Fig. 2a. We are interested in waves reflected from the bottom reflector, see Fig. 2b. The P-wave velocity below the reflecting interface is again \( v_2 \), and in the overburden \( v_1 \), with \( v_1 < v_2 \). We, however, assume that a thin layer with velocity \( v_3 \geq v_2 > v_1 \) is situated in the overburden.

![Diagram](image)

**Fig. 2.** A schematic explanation of the transmission of a P reflected wave through a thin high-velocity layer situated in the overburden. (a) Velocity depth distribution. (b) Subcritical angle of incidence at the thin high-velocity layer. (c) Overcritical angle of incidence at the thin high-velocity layer: Tunneling of energy through the thin high-velocity layer.
The wave field of the wave reflected from the bottom halfspace will obviously be, considerably influenced by the high-velocity thin layer.

If we use the standard ray method for the computation (and do not consider inhomogeneous waves), the amplitude-distance curve of the reflected wave differs considerably from the amplitude-distance curve calculated for a model without the thin transition layer. The main reason for this difference is as follows. As discussed above, the amplitudes of reflected waves are small in the subcritical region \( i_1 < i^*_1 \) and large in the critical and overcritical region \( i_1 > i^*_1 \), where \( i^*_1 \) is given by (1). If \( \alpha_3 \geq \alpha_2 \), no rays with angles \( i_1 \) close to \( i^*_1 \) penetrate through the thin layer; such rays are screened by the thin high-velocity layer. Only rays with angles \( i_1 \) satisfying the relation \( \sin i_1 < \alpha_1/\alpha_3 \) penetrate below the thin layer and strike the bottom reflector. Thus, the critical region related to the reflector with strong amplitudes is completely suppressed by these computations. The calculated amplitudes are, in general, considerably weaker than in a model without the thin layer (as reflected waves correspond only to subcritical reflections). This effect is, however, fully due to the inaccuracy of the standard ray method, i.e. due to its inability to consider inhomogeneous waves. Actually, the inhomogeneous waves carry most of the energy through the thin layer in this case. If we do not use the standard ray method, but some other more sophisticated method which includes computations of inhomogeneous waves, the situation is quite different. For thin transition layers (small \( h/\lambda \)) sufficient seismic energy is tunneled through the thin layer, even for angles of incidence \( i_1 \) with \( \sin i_1 > \alpha_1/\alpha_3 \), see Fig. 2c. We then obtain strong critical reflections from the bottom interface, even though regular rays with angles \( i_1 \) close to \( i^*_1 \) do not penetrate through the layer. The differences between the reflected wave fields computed by the ray method and by any more sophisticated method (e.g., by the hybrid ray-reflectivity method) are very dramatic in this case. Numerical examples will be presented in the next section.

For a detailed discussion of the effects of a thin high-velocity layer on the reflected wave field see [23, 29]. Fuchs and Schulz [23] presented interesting examples of synthetic seismograms for models containing a high-velocity layer, computed by the reflectivity method. The paper also gives many other references related to the effects of a high-velocity layer, both theoretical and observational.

3. NUMERICAL EXAMPLES

In this section, we will study numerically the effects of a thin high-velocity layer, situated in the overburden, on amplitudes and seismograms of reflected \( P \) waves. The hybrid ray-reflectivity method will be used for computations. The results will be compared with standard ray theory computations and with full reflectivity method computations.

The numerical examples were computed using the BEAM87 program package, described in [10]. The program package is designed for hybrid ray-reflectivity
computations in general 2-D laterally varying layered structures, containing a thin transition layer. Instead of the standard ray method, the Gaussian beam summation method can also be optionally used in combination with the reflectivity method. The model may contain isolated bodies, blocks, fractures and pinchouts. The structure within the thin layer is simulated by a system of very thin layers; the velocities and densities in these layers may be arbitrary. Thus, the thin transition layer may represent a laminated layer, a high-velocity layer, an actual transition region with a high gradient of velocity, etc. The thin layer may be smoothly curved and the structure within the thin layer may also slightly vary laterally. Slight absorption can also be considered.

A point source may be situated at any point of the medium, with the exception of the thin layer. Various radiation patterns and source-time functions may be considered. In our computations, we will use isotropic radiation patterns, and a source-time function representing a Gabor signal,

$$x(t) = \exp \left\{ -\frac{(2\pi f_m(t - t_0) / \gamma)^2}{\gamma^2} \right\} \cos \left( 2\pi f_m(t - t_0) + \delta \right), \quad (3)$$

with $f_m = 5 \text{ Hz}$, $\gamma = 4$, $t_0 = 0.4 \text{ s}$ and $\delta = 0$. Note that $f_m$ has the meaning of the prevailing frequency.

We will present three examples of computations. In the first one, the model is one-dimensional, and the thin layer represents a homogeneous high-velocity layer. In the second example, the thin high-velocity layer has a laminated character. In the third example, a 2-D laterally varying structure is considered. The reflecting interface is dipping and the thin layer is curved. The velocity distribution within the individual layers varies both vertically and laterally.

**Thin homogeneous high-velocity layer**

The first model is very simple. It consists of one homogeneous layer, bounded by the plane surface of the Earth at the top, and overlaying a homogeneous half-space at the bottom. The bottom interface (reflector) is placed at a depth of $4.0 \text{ km}$ and is horizontal. The P and S velocities and densities in the layer are $\alpha = 1.5 \text{ km/s}$, $\beta = 0.87 \text{ km/s}$, $\varrho = 2000 \text{ kg m}^{-3}$, and in the halfspace $\alpha = 2.9 \text{ km/s}$, $\beta = 1.15 \text{ km/s}$, $\varrho = 2100 \text{ kg m}^{-3}$. The P-wave point source with an isotropic radiation pattern is situated close to the Earth's surface. The interaction of the source with the Earth's surface is not taken into account. The source-time function is given by (3).

Four alternative computations are performed. In the first version, no thin high-velocity layer is considered, and the standard ray method is used for computation. The ray diagram of the P waves reflected from the reflector is then very simple, see Fig. 3. Ray diagrams of a P wave reflected from a single interface. (a) No thin high-velocity layer is situated in the overburden. (The bold line at a depth of $2.0 \text{ km}$ has only a formal meaning, the interface is fictitious). (b) A thin homogeneous high-velocity layer is situated in the overburden. Standard ray computations. (c) A thin homogeneous high-velocity layer is situated in the overburden. Hybrid ray-reflectivity computations. Note the great difference between (b) and (c).
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Fig. 3a. The bold horizontal line at a depth of 2.0 km does not represent any interface; it merely indicates where a thin high-velocity layer will be inserted in the next three versions.

In the second, third and fourth versions, a thin high-velocity layer, 0.05 km thick, is inserted into the overburden, at depths between 2.0 and 2.05 km. The reflector is then also shifted from 4.0 to 4.05 km. The $P$ and $S$ velocities and density within the thin layer are $\alpha = 2.0$ km/s, $\beta = 1.15$ km/s, $\varrho = 2100$ kg m$^{-3}$. Thus, the medium parameters within the thin layer are exactly the same as in the halfspace below the reflector. Note that the ratio of the thickness of the layer to the prevailing wavelength is 0.125. This number indicates that the hybrid ray-reflectivity method can be applied.

In the second version, the standard ray method is again used for computations, but in a model containing the thin high-velocity layer. The relevant ray diagram is shown in Fig. 3b. As we can see in the figure, the thin high-velocity layer influences the ray diagram considerably. For larger epicentral distances, the ray elements within the thin layer are very close to horizontal. The angle of incidence at the reflector is less than the relevant critical angle, at all epicentral distances. It is obvious that this behaviour of the ray diagram will influence considerably the synthetic seismograms of waves reflected from the reflector situated at a depth of 4.05 km.

In the third version, the hybrid ray-reflectivity method is used for computations, again in the model containing the thin high-velocity layer. The relevant ray diagram is shown in Fig. 3c. The transmission through the thin high-velocity layer is locally evaluated by the reflectivity method. To express this fact graphically in Fig. 3c, normal lines to interfaces are plotted instead of rays through the thin layer; they connect the ray elements which would form the ray if the thin high-velocity layer were removed. They also indicate the tunneling of the inhomogeneous waves through the thin layer.

In the fourth version, the full reflectivity method [22, 30] is used for the computation. The reflectivity zone extends from a depth of 2 km to 4.05 km and contains both the high-velocity layer and the bottom reflector. Thus, the reflectivity code yields not only the waves reflected from the bottom interface at a depth of 4.05 km but also the reflections from the thin high-velocity layer at depths of 2–2.05 km, and the relevant converted waves.

The relevant synthetic seismograms corresponding to the discussed four versions are presented in Figs. 4a, b, c, d. Let us first briefly explain Figs. 4a, b, c, showing synthetic seismograms of the $P$ waves reflected from the bottom interface. Fig. 4a shows the ray synthetic seismograms for the model without the thin high-velocity layer (see Fig. 3a); Fig. 4b the ray synthetic seismograms for the model with the thin high-velocity layer (see Fig. 3b), and Fig. 4c the hybrid ray-reflectivity seismograms for a model with the thin high-velocity layer (see Fig. 3c). In Figs. 4a, b, c, the relevant head waves generated at the bottom reflector and the waves reflected from the thin high-velocity layer are not computed. Moreover, only simple trans-
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missions through the thin layer are taken into account in Fig. 4b; multiple reflections inside the thin layer and under it are not considered. Finally, the reflectivity method computations for a reflectivity zone extending from a depth of 2.0 km to 4.05 km are shown in Fig. 4d. All multiple reflections, converted, inhomogeneous and head waves, etc., are automatically included in these computations.

In all synthetic seismograms, the time axis is oriented down (as is common in seismic prospecting), and the time is shown in milliseconds. The receivers are distributed regularly along the Earth's surface, with a spacing of 0.5 km. The traces are numbered: number 1 corresponds to an epicentral distance of 1.0 km. Thus, the n-th trace corresponds to an epicentral distance of $(n + 1)/2$ km.

Figure 4a displays the well-known ray theory properties of $P$ waves reflected from a plane reflector. The strongest reflections are obtained close to the critical point which is situated at an epicentral distance of 9.07 km (close to the trace 18). At small epicentral distances the amplitudes gradually decrease with increasing epicentral distance and have a minimum at an epicentral distance of 4.5 km. After this, they increase and reach a maximum at the critical distance, and again slowly decrease. Amplitudes are sufficiently strong in the whole range of epicentral distances under consideration. The shape of the signal changes beyond the critical point. (Head waves were not considered in these computations.)

The ray synthetic seismograms for the same $P$ reflected wave, but in a model with a thin high-velocity layer in the overburden, are shown in Fig. 4b. The ray theory computations now yield a quite different picture. They agree roughly with the computations presented in Fig. 4a only with traces 1-14, i.e. for epicentral distances of less than 7.5 km. The relevant angle of incidence is about 40°. Thereafter, the amplitudes remain practically constant at several traces (or slightly increase), but they decrease radically. They practically vanish at traces 20-21, corresponding to an epicentral distance of about 10.5-11 km. The critical region with its strong amplitudes and phase changes has been removed completely. Note that the critical angle equals 41.8°.

The hybrid ray-reflectivity computations are shown in Fig. 4c. At small epicentral distances (at first 14 traces) they agree fully with the previous results shown in Figs. 4a and 4b. This means that the thin high-velocity layer has practically had no influence on the amplitudes of the waves reflected from the bottom reflector, even though the wave has been transmitted twice through it (up-down and down-up). This is, of course, valid only for small angles of incidence and for a very thin layer. For larger epicentral distances, the hybrid ray-reflectivity computations yield all critical region effects. The decrease of the amplitudes beyond the critical region is, however, faster than in the model without the thin layer. Moreover, it can be observed that the high frequencies are gradually filtered out. However, these effects are not very pronounced in Fig. 4c as the thickness of the thin high-velocity layer is small, and the Gabor signal under consideration has a very narrow-band amplitude spectrum. They are more pronounced in the computations with a high-
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Fig. 4d

Fig. 4. Synthetic seismograms of $P$ waves reflected from the bottom reflector for the models shown in Fig. 3. (a) Ray synthetic seismograms for the model without a thin high-velocity layer, see Fig. 3a. (b) Ray synthetic seismograms for the model with a thin high-velocity layer, see Fig. 3b. (c) Hybrid ray-reflectivity synthetic seismograms for the model with a thin high-velocity layer, see Fig. 3c. Note the great difference between (b) and (c). (d) The full reflectivity synthetic seismograms. The first distinct wave corresponds to the wave reflected from the thin high-velocity layer, the second to the wave reflected from the bottom reflector, and the last, weaker waves to the converted waves. A comparison of (c) and (d) shows that the hybrid ray-reflectivity method describes well the tunneling of seismic energy through the thin high-velocity layer at overcritical distances.
Fig. 5b

Fig. 5. Synthetic seismograms of the P waves reflected from the bottom reflector for the models shown in Fig. 3, out with a different inner structure inside the thin high-velocity layer. The thin high-velocity layer consists of three laminas with alternating velocities. (a) Ray synthetic seismograms. (b) Hybrid ray-reflectivity seismograms. The reflections from the thin high-velocity layer are not considered.
velocity layer of larger thickness and with a broad-band seismic signal (not presented here).

Both the ray method and the hybrid ray-reflectivity method are only approximate. To appreciate the accuracy of synthetic seismograms shown in Figs. 4b and 4c, the full reflectivity synthetic seismograms shown in Fig. 4d can be used. Two distinct waves can be observed in the reflectivity seismograms: the first corresponds to the P waves reflected from the thin high-velocity layer, and is of no interest to us here. The second distinct wave corresponds to the wave reflected from the bottom interface, discussed in this paper. The two weak waves arriving later correspond to converted waves and are of no interest to us.

The comparison of Figs. 4b, c, and d shows that the standard ray synthetic seismograms of P waves reflected from the bottom interface completely fail in the whole overcritical region (traces 17–38). This has been, of course, expected as the standard ray method does not consider inhomogeneous waves. On the other hand, the hybrid ray-reflectivity seismograms show surprisingly accurate results, even at large epicentral distances (see traces 30–38). There are, of course, several expected differences between the full reflectivity and hybrid ray-reflectivity seismograms. At small epicentral distances, the reflectivity method yields smaller amplitudes than the hybrid ray-reflectivity method. This difference is caused by the well-known effects of the apparent velocity filtering in the reflectivity code. In the critical region (traces 18–25), we can observe the shift of the maxima of amplitudes of reflected waves in the full reflectivity computations from trace 18 to trace 23. This shift, which is frequency-dependent, is also well-known and has been described in detail in the seismological literature. It has nothing to do with the thin high-velocity layer in the overburden and exists even if the thin high-velocity layer is removed. For more details refer to in [9, 10].

In the conclusion of this section we can say that the standard ray method fully fails in the computation of overcritically reflected waves if a thin high-velocity layer is situated in the overburden. In this case, however, the hybrid ray-reflectivity method yields synthetic seismograms of good accuracy.

**Thin laminated high-velocity layer**

We keep all the conditions of computations the same as in the previous section, with one exception; we change the inner structure of the thin high-velocity layer. The thickness of the thin layer also remains the same, i.e. 0.05 km, but the thin layer is divided into three laminas with the same thickness of 0.0167 km. The medium parameters in the first and the third lamina are the same as they were before in the whole thin layer, but the medium parameters in the middle lamina correspond to the parameters in the overburden.

Figures 5a and 5b shown the synthetic seismograms of the P wave reflected from the bottom halfspace. Fig. 5a shows ray synthetic seismograms, and Fig. 5b the hybrid ray-reflectivity seismograms.
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The results are very similar to those demonstrated in Figs. 4b and 4c. At small epicentral distances (traces 1–14), the synthetic seismograms again practically coincide with those presented in Figs. 4a, b, c. Thus, the inner structure of the thin high-velocity layer does not influence considerably the synthetic seismograms at small epicentral distances. In ray synthetic seismograms (Fig. 5a), the decay of amplitudes at greater epicentral distances is even faster than in Fig. 4b. The reflected

![Ray diagrams](image_url)

Fig. 6. Ray diagrams of a P wave reflected from the bottom reflector in a laterally varying 2-D structure. The thin high-velocity layer situated in the overburden is curved, the bottom reflector is dipping. The velocities inside the individual layers vary both laterally and vertically. (a) Standard ray computations. A shadow zone is formed beyond the epicentral distance of 11 km. (b) Hybrid ray-reflectivity computations. No shadow zone is formed. Note that the rays at larger epicentral distance are curved due to lateral variations of velocity.
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waves practically vanish at traces 18–19. On the other hand, the hybrid ray-reflectivity synthetic seismograms for the high-velocity laminated layer display larger amplitudes at larger epicentral distances than the hybrid ray-reflectivity seismograms for the high-velocity homogeneous layer. Similarly as in Fig. 4c, the signals at larger epicentral distances have a more low-frequency character.

Curved thin high-velocity layer in a laterally varying structure

The model is again similar to the one above. The thin layer, however, is slightly curved and the reflector is slightly dipping. Moreover, the velocities in the overburden vary both laterally and vertically. Even the velocity inside the thin layer is not constant laterally, but varies from 2.0 km/s on the left-hand side to 2.1 km/s on the right-hand side of the model. We wish to demonstrate that the hybrid ray-reflectivity method can be applied without problems even to laterally varying structures. We, however, do not wish to change the previous model completely, so that only slight variations have been introduced.

Figures 6a and 6b show the ray diagrams for the model described above, corresponding to standard ray computations (6a) and to hybrid ray-reflectivity computation (6b). As we can see in Fig. 6a, the thin high-velocity layer again influences the ray diagram considerably. The rays do not even penetrate beyond trace number 21 (epicentral distance 11.0 km), where a shadow zone is formed. This effect is mainly due to the curvature of the thin layer. The ray diagram in Fig. 6b, corresponding to the hybrid ray-reflectivity method does not, of course, display any shadow zone. We thus expect the differences between the ray synthetic and hybrid ray-reflectivity seismograms to be even more pronounced than in Fig. 5.

The synthetic seismograms calculated by the two methods are shown in Figs. 7a and 7b. The ray seismograms behave similarly as the ray seismograms for the two previous models with a thin layer shown in Figs. 4b and 5a. Even though a ray theory shadow zone is formed in the model under consideration and no shadow zone was formed in previous cases, the actual behaviour of the ray synthetic seismogram is very similar. The behaviour of the synthetic seismograms computed by the hybrid ray-reflectivity method (Fig. 7b) is again drastically different from the ray synthetic seismograms.

4. CONCLUDING REMARKS AND DISCUSSION

The hybrid ray-reflectivity method extends the possibilities of the standard ray method considerably. It can be applied to 2-D and 3-D laterally varying layered structures which contain thin transition layers. The inner structure and physical properties within the thin transition layer may vary arbitrarily in the vertical direction. The method was used in [10] to study the properties of waves reflected from such
thin transition layers. The internal structure in the thin transition layer influences the amplitudes and frequency responses of reflected waves considerably. In this paper, the hybrid method is applied to the transmission of reflected waves through a thin high-velocity layer situated in the overburden of the reflector. It is shown that the method removes the difficulties of the standard ray method which is not able to describe properly the tunneling of seismic waves through the thin high-velocity layer along complex ray-paths at overcritical distances.

The accuracy of the hybrid ray-reflectivity computations related to thin high-velocity layer was tested by comparison with the reflectivity computations. It has been found that, in general, the hybrid ray-reflectivity method describes well the tunneling of seismic energy through a thin high-velocity layer. Differences were, of course, observed in the critical region, as the standard ray method was used in the hybrid ray-reflectivity computations presented in this paper. The hybrid Gaussian beam-reflectivity algorithm, which can be optionally used in BEAM87, would remove these differences.

The hybrid ray-reflectivity method can be used in many other applications important in seismology and in seismic prospecting. One such application is now under investigation and has yielded promising preliminary results. The thin layer in this application is situated directly at the top of the model. Another application is related to inhomogeneous waves. We feel it would be possible to incorporate even waves as pseudospherical, P, S, etc., into the routine ray computations in 2-D and 3-D laterally varying layered structures using the hybrid ray-reflectivity code.

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