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# THE COSMIC COINCIDENCE IN BRANS–DICKE COSMOLOGIES*

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Among the suggested solutions to the cosmological constant problem, we find the idea of a dynamic vacuum, with an energy density decaying with the universe expansion. We investigate the possibility of a variation in the gravitational constant as well, induced, at the cosmological scale, by the vacuum decay. We consider an effective Brans–Dicke theory in the spatially flat FLRW spacetime, finding late time solutions characterized by a constant ratio between the matter and vacuum energy densities. By using the observed limits for the universe age, we fix the only free parameter of our solutions, obtaining a relative matter density in the range $0.25 < \Omega_m < 0.4$. In particular, for $Ht = 1$ we obtain $\Omega_m = 1/3$. This constitutes a possible explanation for another problem related to the cosmological term, the cosmic coincidence problem.

*Keywords*: Cosmological constant; scalar-tensor theories.

We are celebrating the 90th birthday of General Relativity, a theory that has survived several precise tests, including verifications of the Equivalence Principle in its weak and strong versions. Such tests generally refer to weak or intermediate gravitational fields, but we have enough reasons to believe that general relativity is also the classical theory of gravitation in the strong field limit.

The theory has then the same theoretical and observational status of Maxwell's electromagnetism. Nevertheless, quantum electrodynamics predicts corrections to its classical version in the presence of very strong fields, corrections related to vacuum effects, as the running of masses and couplings with the energy scale, and the break of the linearity (the principle of superposition) characteristic of classical electrodynamics.

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Despite the absence of a quantum version of general relativity, one can argue about similar vacuum effects in the strong field limit. In this case, a running of masses would be equivalent (at least in what concerns gravitation) to a running of the gravitational constant $G$ (breaking the strong version of the equivalence principle).

Consider, for example, the problem of vacuum energy in cosmology. When we derive the vacuum energy density by means of quantum field theories in flat spacetime, we obtain a divergent result, which must be exactly canceled by a bare cosmological constant in Einstein's equations, because in the flat spacetime the left-hand side of these equations is identically zero. Now, if we calculate the vacuum density in the expanding background, we also obtain a divergent result, but a renormalized value may be derived by subtracting the Minkovskian contribution. The resulting vacuum density will depend on the curvature, and so will vary with time as the universe expands. This mechanism may constitute a possible explanation for the cosmological constant problem,[1,2] leading from a huge value at Planck times to the small value observed nowadays.[3–5]

In the absence of a definite quantum field theory in curved spacetimes, the effects of a decaying vacuum density and of an induced time variation of $G$ may be investigated in the realm of effective geometric theories that respect the energy conservation and the weak equivalence principle. An example is the scalar–tensor Brans–Dicke theory,[6] which will be considered here (for another approach, see Refs. 7 and 8). As "effective" we mean, among other things, that the parameter $\omega$ characteristic of the theory depends on the scale, being very high in the weak field case (leading to unperturbed Einstein equations), but of the order of unity in the cosmological limit. As we are concerned only to this limit, we will use the simplest theory, with constant $\omega$.

Note that we are considering a time variation of $G$ at the cosmological scale, and therefore such a variation cannot be ruled out by local observations. In the case of a stationary spacetime, $G$ or any other dynamical quantity are independent on time. (Even in the context of Brans–Dicke theories, the scalar field $\phi$ has different dynamics at the local and cosmological scales, in the same way as the metric tensor field.)

Consider a spatially flat FLRW spacetime filled with a cosmic fluid composed by dust with energy density $\rho_m$, and by a vacuum term with energy density $\rho_\Lambda$ and pressure $p_\Lambda = -\rho_\Lambda$. This equation of state for the vacuum is a natural choice, since the vacuum has the symmetry of the background. Then, the total energy density is given by $\rho = \rho_m + \rho_\Lambda$, while the total pressure is $p = -\rho_\Lambda$. Under these conditions, the Brans–Dicke field equations can be written as

$$H^2 = \frac{8\pi}{3}\frac{\rho}{\phi} - \frac{\dot{\phi}}{\phi}H + \frac{\omega}{6}\frac{\dot{\phi}^2}{\phi^2}\,, \tag{1}$$

$$\dot{\rho} = -3H(\rho + p) = -3H\rho_m\,, \tag{2}$$

$$\frac{d}{dt}(\dot{\phi}a^3) = \frac{8\pi}{3+2\omega}(4\rho - 3\rho_m)a^3\,, \tag{3}$$

where $H = \dot{a}/a$ is the Hubble parameter.

If $\rho_\Lambda$ is constant, this system is solvable. But, in the case of a decaying vacuum density, we have three equations to determine the functions $a$, $\rho$, $\rho_m$ and $\phi$. Therefore, it is necessary to add some physical constraint, which may be the evolution law for $\rho_\Lambda$, or for $G$, which could be obtained if we had a definite description of vacuum in expanding backgrounds. Here, we will infer an evolution law for $G$ from an observed relation between $G$ and $H$ (usually called Eddington–Dirac or Weinberg relation), given by $G \approx H/m^3$, where $m$ is the energy scale of the QCD phase transition, the latest cosmological vacuum transition. We have presented elsewhere a possible justification for that relation, based on the holographic conjecture.[8–10] Here we will simply postulate that, if it is valid nowadays, it may also be valid at any time of the present phase of universe evolution. Since in Brans–Dicke theory, $G \sim \phi^{-1}$, we then write

$$\phi = \frac{G_0}{G} = \frac{8\pi G_0 \lambda}{H}\,, \tag{4}$$

where $G_0$ and $\lambda$ are positive constants of the order of unity and $m^3$, respectively.

With this constraint one can obtain the general solution of (1)–(3). For our purposes it is enough to find a class of particular solutions describing the late time observed universe. Observations suggest that $\rho_m$ is approximately one third of the critical density $\rho_c = 3H^2/(8\pi G)$ (note that, in the case of Brans–Dicke theories, the flatness of space does not imply $\rho = \rho_c$). On the other hand, the vacuum energy density contributes, at most, with a similar figure, otherwise its effects would be more evident. Therefore, the total energy density is close to $\rho_c$. As before, we will assume that this is not a mere coincidence, but it is a functional relation, valid for any time of the latest phase of the expansion. With the help of (4) we can then write

$$\rho = \frac{3\gamma H^2}{8\pi G} = 3\gamma\lambda H\,, \tag{5}$$

with $\gamma \sim 1$.

Before going on, let us stress that relations (4) and (5) are supposed to be valid during the present epoch, that is, in the limit of late times. The variation law for $G$ (or for the vacuum density) in each era of universe history will depend on the underlying vacuum physics, and is not necessarily the same throughout the whole expansion. Therefore, the model presented here cannot say, in its present form, anything concerning earlier time phenomena like nucleosynthesis, the cosmic background radiation or structure formation.

Substituting the ansatz (4)–(5) in Eq. (1), we obtain an evolution equation for $H$, given by

$$AH^4 - \dot{H}H^2 - B\dot{H}^2 = 0\,, \tag{6}$$

where

$$A \equiv 1 - \frac{\gamma}{G_0}\,, \tag{7}$$

$$B \equiv \frac{\omega}{6}\,. \tag{8}$$

For $A = 0$ (that is, $\gamma/G_0 = 1$), the de Sitter universe is a possible solution. Apart this case, the general solution of (6) is given by

$$Ht = n\,, \tag{9}$$

with

$$n = \frac{\sqrt{1+4AB} - 1}{2A}\,, \tag{10}$$

and where the integration constant was chosen so that the divergence of $H$ occurs at $t = 0$.

Integrating once more we obtain, for the scale factor,

$$a(t) = a_0 t^n\,. \tag{11}$$

The deceleration parameter $q \equiv -a\ddot{a}/\dot{a}^2$ is then given by

$$q = \frac{1}{n} - 1\,. \tag{12}$$

Let us now verify whether this solution satisfies the remaining field equations. Substituting (5) and (9) into (2) leads to

$$\rho_m = \frac{\lambda\gamma}{n} H\,. \tag{13}$$

From this and (5), one obtains the relative energy density of matter,

$$\Omega_m \equiv \frac{\rho_m}{\rho} = \frac{1}{3n}\,. \tag{14}$$

This is an interesting result, meaning that, in the present phase of universe evolution, the relative matter density is constant. This may solve a second problem related to the cosmological constant, namely the approximate coincidence between the matter and vacuum energy densities. Here, such a coincidence follows naturally, without the necessity of any fine tuning.

Note also that our solution depends on just one parameter, the constant $n$ in (9) (usually called the age parameter). So, fixing $n$ by using, say, observational limits for the universe age, we can verify whether the solution agrees with other cosmological parameters, as the relative matter density or the deceleration parameter.

Current astrophysical limits on the universe age lead to an age parameter in the interval $0.8 < Ht < 1.3$ (see, for instance, Ref. 11). Then, from (9) and (14) we obtain $0.25 < \Omega_m < 0.4$. The most probable value $Ht \approx 1$ leads to $\Omega_m \approx 1/3$. On the other hand, from (12) we have $-1/4 < q < 1/4$, the most probable value being $q \approx 0$. The observations still are not sufficiently precise to give the correct value of

$q$. Nevertheless, some authors have recently claimed that, if the scale factor follows a potential law like (11), the best fitting of redshift-distance relations for supernova Ia and for compact radius sources is obtained for $n \approx 1$, that is, for $q \approx 0$.[12–14]

From (4), (5) and (13), it follows, for the vacuum term,

$$\rho_\Lambda = \rho - \rho_m = \left(3 - \frac{1}{n}\right)\gamma\lambda H \sim m^3 H\,, \tag{15}$$

$$\Lambda = 8\pi G\rho_\Lambda = \left(3 - \frac{1}{n}\right)\gamma H^2\,. \tag{16}$$

We obtain a decaying vacuum density, in the spirit of the introduction, with no necessity of postulating a variation law in our original ansatz. Surprisingly enough, the above equations agree with results derived on the basis of quantum field estimations in expanding backgrounds.[15–17] The decaying law (16) has also been heuristically suggested by several authors in both, varying or constant $G$, cases (see, for example, Refs. 18–20).

The reader may note that the matter density (13) does not scale as $a^{-3}$, as it should if the energy of matter were conserved. This is consequence of the conservation of the total energy, expressed by Eq. (2). The vacuum decay is only possible if associated to a process of matter production (a general feature of vacuum states in non-stationary spacetimes). With the help of (9), (11) and (13), it is possible to verify that the rate of production (the relative variation rate of $\rho_m a^3$) is given by $\Gamma = (3 - 1/n)H$. In what concerns the variation of $G$ (at the cosmological scale), from Eq. (4) we have $\dot{G}/G = -(1+q)H$.

Finally, we have to verify Eq. (3). By using (4), (5), (9), (11) and (13), it reduces to

$$4n - 1 = \frac{3 + 12B}{1 - A}\,, \tag{17}$$

where $A$ and $B$ were defined in (7) and (8). The system of Eqs. (17) and (10) allows one to express $A$ and $B$ (that is, $\omega$ and $\gamma/G_0$) as functions of the age parameter $n$. For $n \approx 1$, one obtains $\omega \approx 6/5$ and $\gamma/G_0 \approx 9/5$. Therefore, both $\omega$ and $G_0$ are positive and (since $\gamma \sim 1$) of the order of unity.

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