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ON THE VACUUM ENTROPY AND THE COSMOLOGICAL CONSTANT*

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It is generally accepted that the entropy of an asymptotically de Sitter universe is bounded by the area, in Planck units, of the de Sitter horizon. Based on an analysis of the entropy associated to the vacuum quantum fluctuations, we suggest that the existence of such a holographic bound constitutes a possible explanation for the observed value of the cosmological constant, theoretically justifying a relation proposed 35 years ago by Zel'dovich.

Keywords: Cosmological constant; holographic principle.

As extensively discussed in the literature,[1] there is a fundamental problem related to the existence of a positive cosmological constant. If Λ originates from vacuum energy, its expected value, on the basis of quantum field calculations with a cutoff given by the Planck energy, has the order of $l_{\text{Planck}}^{-2} \approx 10^{70}$ m^{-2}. This is 122 orders of magnitude greater than the observed value $\Lambda \approx 10^{-52}$ m^{-2}.[2] Even considering smaller cutoffs, as the energy scales of electroweak or QCD phase transitions, the expected value is still over 40 orders of magnitude too high. This huge discrepancy is known as the cosmological constant problem.

We have shown elsewhere[3,4] that this problem can be related to other open issues in cosmology, as the large numbers coincidence and the cosmic coincidence problem, with the help of the holographic principle.[5] In this essay, we will try to show that the application of this principle to an asymptotically de Sitter universe leads to a value for Λ in accordance with observation.

In a simplified form, the holographic principle can be described as the extension, to any gravitating system, of the Bekenstein–Hawking formula for black-hole entropy. More precisely, it establishes that the number of degrees of freedom of

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the system is not bounded by its volume in Planck units, as expected from quantum theories of space–time, but by the area, in Planck units, of its delimiting surface.

The use of this principle to the universe as a whole depends on the definition of such a surface at a cosmic scale. The existence of a positive cosmological constant naturally introduces a characteristic surface of radius $\Lambda^{-1/2}$.[6] Another possibility is to use the Hubble horizon, with radius H^{-1}, for it defines the scale of causal connections for any observer.[7] It is clear that in a homogeneous and isotropic, infinite universe, filled with dust and a positive cosmological constant, this last version implies the former, for the Hubble radius tends asymptotically to $\sqrt{3/\Lambda}$. Therefore, in the context of an asymptotically de Sitter universe, the holographic bound establishes that the maximum number of available degrees of freedom is given by

$$N_{\max} \approx \Lambda^{-1} \,. \tag{1}$$

How much entropy has our universe? We know that the number of baryons is of the order of 10^{80}, and that the cosmic background radiation contains about 10^8 photons per baryon. It is reasonable to believe that dark matter contributes with a similar figure. But the major contribution to the entropy of matter seems to come from massive black-holes present in galactic nuclei, which represent an entropy of the order of 10^{101}.[8]

But what about the entropy associated to the vacuum fluctuations? To put this question in a proper way and to clarify its role in solving the cosmological constant problem, let us inquire more carefully on the origin and meaning of this problem.

The difficulties appear when we calculate the vacuum energy density with the help of quantum field theories in flat space–time. The vacuum energy comes from two kinds of contributions. The first one is the energy associated to the vacuum expectation value of self-interacting fields, as the Higgs field, the quark and gluon condensates of QCD, or any other field associated to vacuum phase transitions. The second kind comes from the zero-point fluctuations of the fields, which lead to infinite results. To regulate them, it is used to impose some energy cutoffs, but, as said before, this leads to results still many orders of magnitude too high compared to observation.

Nevertheless, as discussed by some authors (see, for example, Ref. 9), any contribution, infinite or not, to the vacuum energy density predicted by quantum field theories in flat space–time must be exactly canceled by a bare cosmological constant in the Einstein equations, because in the flat space–time the right-hand side of those equations is identically zero. Therefore, to properly pose the problem, we have to calculate the vacuum energy density in a curved background. As before, we find a divergent result as well. But now a physically meaningful (renormalized) value for Λ should be obtained by subtracting the Minkowskian result. Since the space–time of our universe is not strongly curved at cosmic scale, we expect to obtain a small value for Λ, in accordance with observations.

The situation is analog to what occurs in the Casimir effect. There, the zero-point fluctuations of the electromagnetic field give rise to an infinite contribution to the vacuum energy density, inside and outside the region between the Casimir plates. But what is physically meaningful, leading to observable effects, is the difference between the values in the two regions, which is shown to be finite. In our case, the role of Casimir plates is played by gravity.

Calculating the vacuum energy density in a curved background is a difficult task. An example of a rough estimation in the line of the above reasoning was recently given by Schützhold.[9] He estimates the contribution for Λ from the chiral anomaly of QCD in a curved, expanding space–time, obtaining $\Lambda \approx H\Lambda_{\rm QCD}^3$, where $\Lambda_{\rm QCD}$ is the energy scale of the chiral phase transition. In the limiting case of a de Sitter universe, $H \approx \sqrt{\Lambda}$, and his result leads to

$$\Lambda \approx m^6 \tag{2}$$

(where we have made $m = \Lambda_{\rm QCD}$).

This expression was derived 35 years ago by Zel'dovich, from empirical arguments.[10] Although it is sensible to the parameter m, (2) leads to the correct order of magnitude: using $\Lambda_{\rm QCD} \approx 150$ MeV, we obtain $\Lambda \approx 10^{-51}$ m^{-2}, in good agreement with observation, considering that it was not taken into account numerical factors.

An alternative to circumvent the difficulties involved in quantum field calculations in a curved background is to use a thermodynamic approach, which does not depend on the details of the field dynamics. The idea is to obtain a superior limit for the vacuum entropy, instead of its energy density, and to compare the result with the holographic bound (1). The reader may argue that it is not trivial to define the number of virtual particles in curved backgrounds. Let us remind, however, that our universe has a quasi-flat space–time. Therefore, the estimation given below can be considered a good approximation.

It is clear that non-trivial vacuum configurations of classical fields (as the vacuum expectation value of the Higgs field or the QCD condensates) do not contribute to the vacuum entropy. In what concerns the zero-point fluctuations, they have, properly speaking, an infinite entropy density, because (if we do not impose any energy cutoff) the number of modes is infinite. But if we regulate their energy, by introducing an ultraviolet cutoff m, we also regulate their entropy. A simple estimation of the resulting entropy bound can be derived as follows.

Limiting the energy–momentum space associated to the zero-point fluctuations leads to the quantization of their configuration space, with a minimum size given by $l \approx m^{-1}$. This results in a superior bound to the number of available degrees of freedom in a given volume, say, the volume inside the Hubble horizon. The maximum number N of observable degrees of freedom will be of the order of V/l^3, where V is the Hubble volume. That is,

$$N \approx \left(\frac{m}{H}\right)^3 . \tag{3}$$

Now, if we take for H the de Sitter asymptotic value $H \approx \sqrt{\Lambda}$, we can identify (3) with the holographic bound (1). It is easy to verify that this leads to Zel'dovich's relation (2).

But why m coincides to the energy scale of the QCD phase transition? The common belief is that a natural cutoff should be given by the Planck energy, for at the Planck scale the classical picture of space–time breaks down. Nevertheless, it is not difficult to see that, equating (1) to (3), with $H \approx \sqrt{\Lambda}$ and $m = m_{\mathrm{Planck}}$, one obtains a Hubble radius of the order of l_{Planck}, which is not consistent with our universe.

One can also argue that the zero-point fluctuations of other fields than quarks and gluons contribute to the entropy as well. It is then intriguing that just Λ_{QCD} enters in Zel'dovich's relation. Note, however, that in a curved space–time the different sectors of the standard model of particles interactions are coupled by gravity. On the other hand, the de Sitter universe is a stationary space–time, and, therefore, all the (interacting) vacuum fields should tend to a state of thermodynamic equilibrium, at the temperature of the last vacuum phase transition. But the last of such transitions was the chiral transition of QCD, at a temperature given by Λ_{QCD}.

Finally, let us note that the vacuum entropy already dominates the universe entropy. Indeed, taking for H the value observed nowadays, $H \approx 65$ km/(sMpc),[2] we obtain from (3) (with $m \approx \Lambda_{\mathrm{QCD}}$) $N \approx 10^{122}$, a figure that predominates over the matter entropy referred above, of order 10^{101}.

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References

1. S. Weinberg, *Rev. Mod. Phys.* **61**, 1 (1989); V. Sahni and A. Starobinsky, *Int. J. Mod. Phys.* **D9**, 373 (2000); S. E. Rugh and H. Zinkernagel, *Stud. Hist. Philos. Mod. Phys.* **33**, 663 (2002); N. Straumann, astro-ph/0203330; A. D. Dolgov, hep-ph/0203245; T. Padmanabhan, *Phys. Rep.* **380**, 235 (2003).
2. M. Tegmark, M. Zaldarriaga and A. J. S. Hamilton, *Phys. Rev.* **D63**, 043007 (2001); X. Wang, M. Tegmark and M. Zaldarriaga, *Phys. Rev.* **D65**, 123001 (2002).
3. G. A. Mena Marugán and S. Carneiro, *Phys. Rev.* **D65**, 087303 (2002).
4. S. Carneiro, gr-qc/0206064.
5. G. 't Hooft, gr-qc/9310026; L. Susskind, *J. Math. Phys.* **36**, 6377 (1995). See also T. Padmanabhan, *Gen. Rel. Grav.* **34**, 2029 (2002); T. Jacobson and R. Parentani, *Found. Phys.* **33**, 323 (2003).
6. R. Bousso, *J. High Energy Phys.* **0011**, 038 (2000); *Rev. Mod. Phys.* **74**, 825 (2002).
7. G. Veneziano, *Phys. Lett.* **B454**, 22 (1999); R. Easther and D. Lowe, *Phys. Rev. Lett.* **82**, 4967 (1999); N. Kaloper and A. Linde, *Phys. Rev.* **D60**, 103509 (1999).
8. R. Penrose, *The Emperor's New Mind* (Oxford University Press, 1989), Chapter 7.

9. R. Schützhold, *Phys. Rev. Lett.* **89**, 081302 (2002); *Int. J. Mod. Phys.* **A17**, 4359 (2002). The relation between chiral anomaly and vacuum energy density is also discussed in M. Novello, J. Barcelos-Neto and J. M. Salim, *Class. Quantum Grav.* **18**, 1261 (2001); **19**, 3107 (2002).
10. Ya. B. Zel'dovich, *JETP Lett.* **6**, 316 (1967); *Sov. Phys. Uspekhi* **11**, 381 (1968). Zel'dovich used for m the proton mass. The idea associating m to the energy scale of the QCD phase transition can also be found in R. A. J. Matthews, *Astron. & Geophys.* **39**, 19 (1998).